INTRODUCTION

In recent years, there has been a rapid adoption of mobile devices in educational institutions worldwide. The hegemony and dominance of fixed personal computers is diminishing quickly, making way for a variety of flexible, ubiquitous alternatives that have enormous implications for learning, within both formal and informal settings. We are in the midst of the timeframe where tablets and smartphones are becoming standard learning tools, and adopting mobile devices in the classroom and beyond is becoming a must (Johnson et al., 2013). Mobile mathematics learning – which studies the ways in which the mobility of learners, augmented by suitable handheld devices, can contribute to the process of gaining new mathematical knowledge, skills and experiences – has become a cutting-edge research area that has lately attracted considerable attention by the mathematics education community.

Despite the increased interest on mobile devices as learning tools, the amount of available primary research studies on their integration into mathematics teaching and learning is still relatively small due to the novelty of these technologies. Research on the actual impact of mobile technologies on mathematics learning outcomes is still scarce and its findings inconclusive at this point. However, the conducted studies do overwhelmingly point towards numerous positive attributes that have the potential to enhance both formal and informal learning of mathematics (e.g. McKenna, 2012; Clark & Luckin, 2013). There is considerable convergence in research findings which highlight significant benefits of tablets and other mobile devices. Research suggests that the adoption and use of personalized devices within and beyond the classroom confines, can provide an educationally rich and dynamic environment that increases students’ autonomy, allowing them to augment and enhance deep, hands-on learning in ways previously not possible or not so easy to do (Clark & Luckin, 2013; Burden et al., 2012). The portability, speed, simplicity of interface, and accessibility of mobile devices makes them very easy to use by all involved in the educational process, including very young children, non-tech-savvy parents and teachers, and students with disabilities (Melhuish & Falloon, 2010). The ever-growing list of interactive learning tools and applications available, and the potential for personalised and seamless learning, provides instructional designers and teachers with more options for creating effective learning experiences that move beyond static presentation, limited interaction, and the walls and schedules of formal schooling (Clarke & Svanaes, 2012; Burden et al., 2012).

While mobile devices present some exciting opportunities for a transformative shift in mathematics education (Melhuis & Falloon 2010) their introduction into educational settings does not come without challenges. The existing literature highlights not only opportunities, but also a number of pedagogical, technical, and management issues that need to be addressed for their successful integration within exist-
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ing school systems and beyond. The adoption of mobile learning brings huge challenges to the teaching and learning culture and, to be successful, it requires major changes to traditional forms of education. As several studies have indicated, when tablets and other mobile devices enter classrooms and other learning environments, their impressive immersive capabilities are often overlooked or underdeveloped, and they are used in very traditionally structured stand-and-deliver classroom settings (Daccord, 2012).

For mobile technologies to be more effectively utilized to enhance mathematics learning opportunities for all learners, there needs to be a re-conceptualization of the design and management of learning environments. Careful strategic planning and reflective implementation, grounded in solid research, is necessary.

EDITED VOLUME OBJECTIVES

The urgent need for more scholarly publications investigating the infiltration of mobile devices into mathematics teaching and learning practices and experiences, has led to the conception of the idea for developing an edited volume on mobile mathematics learning. The mission of this publication is to inform best practices in mathematics education research and teaching practice by providing an account of current and future trends and issues in mobile mathematics learning, and associated technologies and educational methodologies. Specifically, the book was perceived with the following objectives:

1. To provide an overview of up-to-date international research work on the adoption of mobile technologies in formal and informal mathematics education contexts;
2. To identify and publish worldwide good practices in the design, development, and educational use of mobile technologies in support of students’ mathematics learning processes and outcomes;
3. To contribute to future research and theory-building by addressing theoretical, epistemological and methodological considerations regarding the design of mobile-assisted mathematics learning environments;
4. To account for issues of equity and diversity in mobile mathematics pedagogy, in order to increase participation of groups of learners at special risk of exclusion from mobile mathematics learning.

The edited volume approaches an audience that is broad enough to include all researchers and practitioners interested in the exploitation of mobile technologies in mathematics teaching and learning. Mathematics teachers at all levels of schooling, and in adult education, can get a compilation of best practices and recommended processes for effectively utilizing the vast capabilities of mobile technologies in the mathematics classroom. Researchers interested in exploring and advancing the field of mobile-assisted mathematics learning, can be informed about the latest developments in the field, and about relevant research projects currently being implemented in various formal and informal educational settings worldwide. Educational leaders (e.g. school district administrators, technology coordinators etc.) can get practical advice on how to cope with the administrative and organizational issues surrounding the integration of mobile devices in mathematics teaching and learning. Academic experts, learning technologists, and educational software developers, can get more sensitized to the needs of mathematics learners and their teachers, supporting the development of new methodologies and technological tools which make constructive use of mobile devices. National and transnational education authorities respon-
sible for setting mathematics curricula and educational policies can get useful information regarding current developments and future trends in mathematics education. Finally, teacher training institutions can utilize the book for further improvement of their teacher preparation programs.

EDITED VOLUME CONTENTS

The edited volume has compiled a collection of knowledge on the latest developments and approaches to mobile mathematics learning. It has collected incisive contributions from leading researchers and practitioners internationally on how to best deploy mobile technologies to enhance mathematics content and processes and make them more accessible and relevant to all learners. Contributions present diverse mobile technological tools and applications suitable for mathematics teaching and learning, and document their development and implementation in a variety of educational settings at different levels of schooling, as well as in informal learning contexts. The contributions address both technological and pedagogical issues related to mobile mathematics learning. They identify affordances and challenges arising from the design and educational use of mobile devices, and provide recommendations for future research and practice. The majority of the chapters report on original empirical studies which demonstrate validated practical experiences related to mobile-assisted mathematics learning. Chapters presenting interim results from innovative, ongoing projects have also been included. The volume also contains conceptual essays which contribute to future research and theory-building by presenting reflective or theoretical analyses, or forecasting of emerging mobile learning technologies and tendencies.

The book includes 13 chapters that cover a broad range of topics on the design and application of mobile technologies in a variety of both formal and informal mathematics learning contexts, ranging from schools to the workplace. The book chapters have been organized into four sections covering the following themes: (a) learning and learner support in mobile-assisted practice; (b) pedagogical and instructional design considerations; (c) mobile mathematics and teachers’ training; and (d) practical experiences and ideas for mobile mathematics learning. Each chapter includes extensive bibliographic references for further reading.

Section 1: Learning and Learners Support: Individual, Collaborative, and Situated Mobile-Assisted Mathematics Learning

Section 1 includes chapters which explore issues pertaining to learner and learning support from both research and innovative practice perspectives.

In Chapter 1, Theodosia Prodromou, Maria Meletiou-Mavrotheris, and Andreas O. Kyriakides, report on the main experiences gained from a study which exploited the affordances of tablet devices for sparking young students’ (11-year old) interest in mathematics. Using the general purpose Recce-London App as an exploration tool, children in their study engaged in authentic mathematical problem solving activities related to map navigation. Findings indicate that although students used the Recce-London App in different ways, and adopted different patterns of reasoning, at different levels of sophistication, they all gained valuable experiences from experimenting with the app. The accurate virtual representation of London provided an immersive environment that promoted the application of important mathematical
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Concepts and skills in authentic contexts. The impressive graphics and detail provided by the application, brought London to life and served as a powerful starting point for students to engage in genuine mathematical problem solving, in a collaborative, learner-centered learning environment.

In Chapter 2, Erin Ottmar, David Landy, Erik Weitnauer, and Rob Goldstone present _Graspable Mathematics (GM)_ , a novel approach to algebra instruction they have developed which applies touch screen interfaces to align the content of algebraic transformations with core cognitive and perceptual systems. The authors provide an overview of the process of their touch screen technology development, beginning from basic cognitive research and resulting in an applied educational intervention for algebra. They describe many of the methodological, theoretical, and practical considerations that guided their iterative design and development, and discuss many of the challenges and lessons learned when implementing the intervention in classroom settings.

The next chapter (Chapter 3), by Nathalie Sinclair and David Pimm, describes _TouchCounts_ , a novel iPad application that makes full use of its multi-touch affordance to engage young children in aspects of counting and adding/subtracting, by means of engagement with the combined sensory modalities of the visible, the audible and the tangible. Drawing on various excerpts with children ages three to six working with this App in educational settings (both day-care and kindergarten), the authors engage in an examination and discussion of how this trio of senses is utilised in children’s activity with TouchCounts. By means of exploring the notions of myth and rite in concert with their empirical data from _TouchCounts_ , they offer a tentative alternative account of number sense development, one that increases attention to issues of ordinality, as well as highlighting the significance of tangibility in the context of young children coming to terms with counting and early arithmetic.

Chapter 4, authored by Teija Vainio and Tanja Walsh, presents an overview of a large-scale (n=3500) longitudinal study of learners in Grades 9 and 10, which investigated key cross-cultural design issues in the adoption of mobile technologies in formal and informal mathematics education in two different cultural contexts (Finish and South African). Based on their experiences, the authors argue that cross-cultural design of technology in the context of mobile learning can help to identify culturally sensitive areas such as attitudes towards informal and collaborative learning and recognition of the local context for the content. Based on their experiences, they propose a number of culturally sensitive areas related to concept and content as focus areas of best design practices for designers and developers of mobile learning services.

In Chapter 5, Susan Staats, David Ernst, Shelley Berken and Douglas Robertson, use a reflective practice standpoint to discuss the use of general purpose, iPad-based video production apps to conduct mathematical inquiries in a college algebra class targeting pre-service primary school teachers. Their study comes from a multifaceted four-year project at their university, which is one of the largest tablet initiatives in the world. Using sketching apps to layer photos with grids, and capturing mathematical notation as it was written by, students, the study described mathematical features of artwork that cannot be physically touched for measurement. Reflective commentaries on the context of iPad use from the viewpoints of an administrator and a teacher educator who taught the algebra students in a connected education class were also included in the analysis. Staats et al. discuss the potentials and pitfalls of such a video assignment to uncover the mathematics of these “inaccessible” art objects. Student math videos are described through three themes: choice of video platform; level of engagement with mathematics; and videos as a lens to perceive mathematical thinking. Findings suggest that the use of video and sketching apps leveraged learning across academic and informal learning and supported student mathematical agency.
Chapter 6, authored by Andreas O. Kyriakides, Maria Meletiou-Mavrotheris, and Theodosia Prodromou argues for the importance of embedding computer programming into existing mathematics curricula through the use of educational games apps. It accounts on the main experiences gained from a study which incorporated A.L.E.X., an educational coding game app, within the primary school mathematics curriculum. The study took place in a public school, located in a rural area of Cyprus. The majority of the school’s students come from families with a low socioeconomic status and income. The researchers knowingly selected such a context to orchestrate a teaching intervention. Their goal was to explore the potential of tablet technologies for providing students with knowledge, skills and confidence in doing mathematics. Among the school community, a group of 10-11 year old students were randomly selected to compose the study sample. Findings suggest that while working with A.L.E.X., children identified and processed mathematical themes that emerged spontaneously and experienced unique emotions of surprise and enthusiasm about the existence of games with mathematical content, whereas at the same time they acknowledged the pedagogical role that tablet devices could play in mathematics.

Section 2: Pedagogical and Instructional Design Considerations

Chapters included in Section 2 focus on theoretical, epistemological and methodological considerations related to the design of mathematical learning activities with mobile technologies.

Chapter 7, authored by Pierre Clanché, Antonín Jančařík, and Jarmila Novotná, focuses on the study of the differences between off-line communication using mobile devices, and common communication in conventional settings. Clanché et al. point out the fact that classical communication models do not always reflect the dynamics of communication using mobile devices and other technological tools, and stress the need for examining more closely the different forms of discourse engendered by different forms of technology, and the ways in which this is linked to new kinds of mathematical understandings. They introduce two basic theoretical principles related to teacher’s provision of effective support - the principle of charity and the principle of scaffolding - and explain how these principles should be modified for the unique environment of an off-line discussion. The presented theoretical constructs are illustrated by examples from an online mathematical course targeting secondary pupils. The role of technological devices in developing the quality of pupils’ participation in the course is documented.

Chapter 8, by Vani Kalloo and Permanand Mohan, explores some of the key issues pertaining to the development of mathematics content for mobile learning. The authors first point out the main difficulties that might be encountered in the process of translating mathematics content into content for a touch screen device. They discuss the following key factors that ought to be taken into consideration when designing content for learning mathematics via a mobile device: diversity of mathematics content (text, numbers, algebraic equations, graphs, matrices, etc.), technical limitations of mobile devices as compared to desktop computers, and the need for basing the design on pedagogically sound principles of mathematics learning. Next, drawing upon their personal experience with the development of mobile mathematics learning content, the authors present a method for designing this content in motivating and meaningful ways that can attract learners’ interest and promote learning. An example of a learning activity developed using this method is also presented.
Section 3: Mobile Mathematics and Teachers’ Training

Section 3 focuses on how to support teachers and other educators in the adoption of the new pedagogical approaches that are needed for successfully moving mathematics education into the mobile learning era.

In Chapter 9, Catherine Attard alerts us to the fact that the fast pace of technological advances has meant that new mobile technologies are often being introduced into classrooms without adequate knowledge of how they can be effectively used to enhance teaching and learning in mathematics. To support her argument, she uses data obtained from two studies conducted in Australian primary classrooms to describe how a small group of teachers used a (then) new technology, the iPad, to teach mathematics within the first six months of its implementation and without the support of professional development. The practices of these teachers, including the issues and challenges they experienced and examples of their teaching with iPads, are presented against a backdrop of the SAMR model (substitution, modification, augmentation and substitution) (Puentedura, 2006), and are used in conjunction with the TPACK framework (Koehler & Mishra, 2009) to organise, present and analyse the observed uses of iPads. Findings illustrate the need, prior to the introduction iPads for teaching and learning, of appropriate professional development that addresses the combination of mathematical content, pedagogy and technology that is critical for all teachers, regardless of teaching experience. The chapter concludes with a small sample of mathematics tasks that incorporate creative uses of the iPad use in primary classrooms.

In Chapter 10, Jane Lane shares some of the findings from the Technological, Pedagogical and Content Knowledge (TPACK) iPads in School Project, a large study undertaken over two years in ten West Australian classrooms from kindergarten to year ten, which aimed to investigate how touch enabled mobile devices (iPads) were used in Australian classrooms. A community of practice model supported teacher reflection “on action” and “in action” aligned to national standards. The study used a combination of qualitative and quantitative research techniques, which included self-tracking video devices that gathered evidence on how teachers were using iPads in teaching. Findings from the story revealed a wide range of applications and uses of touch enabled mobile devices in mathematics education, which in some instances redefined teaching and learning with significant benefits for teachers and students. The touch enabled mobile device provided opportunities for teachers to design a personalized curriculum, and for students to engage and to be actively involved in developing or constructing their understanding, which enhanced their learning. At the same time, however, teachers in the project indicated the need for more support and professional learning to enable them to shift their pedagogy beyond a substitution level when using the iPad. It was noted that there needs to be commitment and a shift in thinking by the teachers to change from a traditional teacher-centred approach in teaching mathematics to one in which the students have a central role.

Section 4: Practical Experiences and Ideas for Mobile Mathematics Learning

Section 4 includes chapters which are either conceptual essays contributing to future research and theory building or reports of practical experiences related to mobile-assisted mathematics learning. The authors provide helpful suggestions and strategies on how to address some of the challenges arising from the adoption of mobile learning devices, and/or on how to best utilize the mobile devices’ potential to promote effective learning of mathematics for all learners, including children with disabilities.
Chapter 11, by Howard P. Edwards, focuses on statistics education. It discusses the needs of a Statistics app user and how these differ from the needs of users of other Mathematics apps, and provides a review of a range of apps for mobile devices currently available for use in teaching and learning in the areas of probability and statistics at the undergraduate introductory statistics level. Each app was classified by the author into one of five categories, according to the main purpose for which it was designed. In each category, examples of good and not-so-good apps are provided as an indication of the variability in quality. Findings from the classification indicate that although there is a wide range of apps available that enable users to carry out statistical investigations and to learn about probability and statistics, these apps vary greatly in their levels of usability and quality, with many apps containing faults and errors in their design and/or output. Based on these findings, the chapter raises several issues for consideration by potential users of statistics apps. In particular, it points out the need for potential users to try to obtain independent advice regarding the scholarly content of an app and what it purports to do and to try out the interface (using a lite version if necessary) before deciding whether to purchase and/or to use it.

Chapter 12, authored by Stuart Cork, discusses the potential of touch-enabled mobile devices as vessels for mathematical microworlds, and draws conclusions regarding the influence that such microworlds can have on a learner’s mathematics learning. The first section of the chapter provides an overview of the literature relating to linear equations and microworlds. Insights into the current knowledge of both areas facilitate an analytical framework that is later applied to two case studies of young pre-secondary school learners, with no prior formal teaching on the topic, exploring linear equations through the algebra-based microworld DragonBox, a touch-enabled mobile app available on all major platforms. Findings obtained from the study indicate that young learners can become competent at linear equations using DragonBox in a mobile learning setting. However, DragonBox as an isolated tool proved insufficient for developing a relational understanding of linear equations. Although learners in this study demonstrated procedural fluency when working with DragonBox, it was only through encouraged reflection that a relational understanding seemed to take effect and without it the children developed an algorithmic approach and inconsistent notions about the structure of the microworld. This lack of reflection amongst learners was consistent with the play paradox (Hoyles and Noss, 1992) according to which, while learning is inevitable when working with a microworld; it might not be the intended learning of the designer, teacher, or researcher. Cork concludes that relying on a touch-enabled microwold as an isolated learning tool might lead to procedural fluency rather than conceptual understanding, and that if such type of app were to be incorporated into a curriculum or scheme of work, it would be important to encourage students to reflect in order to develop their relational understanding.

In the final chapter of the edited volume (Chapter 13), Georgios Stylianou and Katerina Mavrou discuss the potential of Indoor Positioning and Indoor Navigation (IPIN) systems in teaching and training Orientation and Mobility (O&M) to individuals with visual impairment. An attempt is made to investigate the implications of such trainings and technology in mathematics education, and, particularly, to explore how the understanding of basic geometrical ideas via orientation training can help to improve mathematical learning, initially within the visually impaired community and at a later stage within the general student population. An IPIN system already developed by Stylianou is first presented, and the additional features that will be incorporated to this system in order to support O&M training are laid out. Finally, the chapter provides some suggested scenarios of the IPIN system’s use in O&M trainings and O&M activities which can be used to introduce and strengthen opportunities for exploring mathematical topics.
CONCLUDING REMARKS

It is our conviction that the embracing of mobile technologies has the potential to truly transform education, through the creation of engaging, aligned with the 21st century, environments that enrich mathematics learning. However, despite mobile devices’ educational potential, the evidence base to support their use as teaching and learning tools in mathematics is scarce. Thus, by focusing on this important emerging area of both research and academic practice, this publication fills a significant gap in the mathematics education literature. To the best of our knowledge, this is the first international book to provide a comprehensive survey of the opportunities and challenges presented by mobile technologies for mathematics teaching and learning. Although a growing number of manuscripts generically cover the topic of mobile learning, no book focuses specifically on the disciplinary particularities of mathematics mobile learning. Similarly, while there are several books on technology-enhanced mathematics education in general, this book provides the first detailed account of the theory and research underlying mobile mathematics learning. With contributions from leading international experts, the book provides valuable insights into contemporary and future trends and issues related to mobile mathematics learning, informing best practices in mathematics education research and teaching practice.

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