Since the 1980s, both the practice and the teaching of mathematics have changed. Vast increases in computational power and the accessibility of data have had a dramatic impact on mathematics, greatly expanding its reach. This book examines a central tool in this expansion—dynamic visualization.

Computation has revolutionized mathematical research. It has brought new tools and opened up new areas of application. Explicit computation can now be the most efficient way to verify a conjecture. Visualizations can illuminate mathematical behavior—such as fractals and chaos—leading to new conjectures. Biology, with its myriads of variables, is finally accessible to quantitative analysis. Engineering now has graphical tools at its disposal, and finance has provided a new arena for mathematical modelers. Computer simulation enables drugs to be designed outside the lab. Climate scientists can provide a range of computer-generated scenarios for the planet’s future. There is little doubt that computation has changed mathematical research permanently.

With changes in research come changes in the teaching of mathematics. First, topics change. Students going into biology, or finance, or climate modeling, for example, want to learn the mathematics used in their disciplines. They need to understand the mathematics in context—how components of the field are represented in the model, and how changing parameters changes the variables of interest.

At the same time as the changes in computation came changes in communication. The Internet and mobile phones have made data widely accessible. Sales on the Web have put many retailers out of business; data on the Web has made students impatient with contrived applications. Online courses have provided educational access where there was none, and are set to be a central medium of the future.

Changes in computation and communication are poised to affect mathematical pedagogy as profoundly as they have affected mathematical research. Traditionally, one method dominated—“chalk and talk”—and the language used was largely symbolic. Calculus became powerful because of its ability to encapsulate the solution to a problem efficiently in symbols. Thus, for generations students learned mathemat-
ics from symbols. Unfortunately, conceptual understanding did not always flow from the symbols. Some students learned to compute, but did not know what they had computed, or how the answer would change if the inputs changed. Classroom norms emerged that equated understanding with symbolic computation. For many students, the phrase “you see that…” came to mean “you know the next step is.”

Traditionally, graphical reasoning was avoided by many students because graphs had to be made by hand. Instructors noticed this reluctance when grading student work—unless points were specifically assigned for a graph, many students did not draw one. Many students considered graphing by hand a burden that was not worth the benefits it delivered to understanding.

The arrival of software and graphing calculators in the 1990s tipped the balance. The accessibility of graphs encouraged more students and teachers to use them. At the same time, statistical software was developed to do the number crunching required by real data. This and the ability to make statistical plots enabled exploratory data analysis to be taught in a way that is closer to professional practice. Calculus and differential equations have benefited from powerful new Computer Algebra Systems (CAS) and from increased visualization using software and applets. Thus, the pedagogy in many college-level mathematics courses has been affected by the power of computation, often by creating visualizations.

Not surprisingly, these changes have also generated concerns. The most frequent are about a reduction in computational skill and a loss of familiarity with abstraction. Many mathematicians believe there has been a widespread decline in computational skill; the data suggests that manipulative skills of students who use technology are about the same as the skills of those who do not.

The link between abstraction and visualization is not well understood; this book starts to fill that gap. As one of the defining characteristics of mathematics and one that gives it much of its power, abstraction is of central importance to mathematicians. Yet little is known about how students develop an understanding of abstraction. Everyday teaching suggests that symbols often do not lead students to understand abstraction. However, since many people think visually, there is good reason to believe that visualization will provide an additional, and perhaps effective, bridge to abstraction. The dynamic nature of the visualization may help students take the step from seeing a static connection to seeing the impact of changing variables.

Much of the development of visualization-driven teaching has been done without a grand plan and without systematic reflection. As new software emerged, new ideas for teaching emerged and were tried. Some survived and became part of accepted practice; others died. Probing the relationship between visualization and abstraction, this book is a welcome and timely contribution to the field.
With chapters describing the use of dynamic software in a range of fields, the book gives readers an overview of the possibilities for its use. Most important, this book invites readers to take the next step and investigate the link between visualization and abstraction.

Deborah Hughes Hallett
Cambridge, MA, USA
June 2013

Deborah Hughes Hallett is Professor of Mathematics at the University of Arizona and Adjunct Professor of Public Policy at the Harvard Kennedy School. With Andrew M. Gleason at Harvard, she organized the Calculus Consortium based at Harvard, which brought together faculty from a wide variety of schools to work on undergraduate curricular issues. She is regularly consulted on the design of curricula and pedagogy for undergraduate mathematics at the national and international levels and has written several college level mathematics texts. She has co-authored a report for the National Academy of Science’s Committee on Advanced Study in American High Schools and is a member of the MAA Committee on Mutual Concerns and chair of the College Board’s Committee to review the Math-SAT. In 1998 and 2002, she was co-chair of International Conference on the Teaching of Mathematics in Greece, attended by several hundred faculty from about 50 countries. In 2006, she chaired the third conference in this sequence in Istanbul Turkey. She has established programs for master’s students at the Harvard Kennedy School, pre-calculus and quantitative reasoning courses (with Andy Gleason), and courses for economics majors. She was awarded the Louise Hay Prize and elected a fellow of the American Association for the Advancement of Science for contributions to mathematics education. Her work has been recognized by prizes from Harvard and the University of Arizona, and as national winner of the MAA Haimo Award for Distinguished Teaching.