Preface

Multirate signal processing techniques are widely used in many areas of modern engineering such as communications, image processing, digital audio, and multimedia. The main advantage of a multirate system is the substantial decrease of computational complexity, and consequently, the cost reduction. The computational efficiency of multirate algorithms is based on the ability to use simultaneously different sampling rates in the different parts of the system.

The sampling rate alterations generate the unwanted effects through the system: spectral aliasing in the sampling rate decrease, and spectral images in the sampling rate increase. As a consequence, the multirate processing might produce unacceptable derogations in the digital signal. The crucial role of multirate filtering is to enable the sampling rate conversion of the digital signal without significantly destroying the signal components of interest. The multirate filtering makes the general concept of multirate signal processing applicable in practice.

This book is focussed on multirate filters, the essential processing algorithm in multirate systems. The mission of the book is to bridge the existing gap between the multirate filter theory and practice. This book deeply introduces MATLAB® functions and commands in presenting and explaining various aspects of multirate filtering. MATLAB® is chosen as the most popular software widely used at universities, in research laboratories, and in industry.

A multirate filter can be defined as a digital filter in which the sampling rate of the input signal is changed in one or more intermediate points. Multirate techniques are used in filters for sampling rate conversion where the input and output rates are different, and also in constructing filters with equal input and output rates. The basic roles of multirate filtering in modern signal processing systems go in three main directions.

Firstly, the multirate filtering is used whenever two digital systems with different sampling rates have to be connected. Filtering is used to suppress aliasing in decimation, and to remove imaging in interpolation. The use of an appropriate filter enables one to convert a digital signal of a specified sampling rate into another signal with a target sampling rate without destroying the signal components of interest.

Secondly, the multirate filtering is one of the best approaches for solving complex filtering problems when a single filter operating at a fixed sampling rate is of significantly high order and suffers from output noise due to multiplication round-off errors and from the high sensitivity to variations in the filter coefficients. Various multirate design techniques provide that the overall filtering characteristic is shared between several simplified subfilters operating at the lowest possible sampling rate. Design constraints for subfilters are relaxed if compared to a single-rate overall filter. As a consequence of the reduced design constraints, the effects of quantization in subfilters and in the overall multirate filter are decreased. Multirate filters provide a practical solution for digital filters with stringent spectral characteristics that are very difficult to solve otherwise.

Third, multirate filtering is used in constructing multirate filter banks.

For multirate filters, FIR (finite impulse response) or IIR (infinite impulse response) transfer functions can be used for generating the overall system. The selection of the filter type depends on the criteria at hand. An FIR filter easily achieves a strictly linear-phase response, but requires a larger number of operations per output sample when compared with an equal magnitude response IIR filter. The linear-phase FIR filter is an adequate choice when the waveform of the signal has to be preserved. An advantage of the multirate design approach is the ability of improving significantly the efficiency of FIR filters thus making them very desirable in practice.

The multirate signal processing and multirate filtering have been attracted many researchers during the last several decades. The rapid development of the new algorithms and new design methods has been influenced by the advances in computer technology and software development. Although the existing literature on the subject is very large, the multirate signal processing is an open area of research.

The multirate filtering is an area of interest for many researchers and practicing engineers. Efficient and sophisticated design in the field of multirate filtering needs a high-level software tool such as MATLAB®. The adequate software enables one to use the built-in functions and algorithms and concentrate on his/her one task (or research problem).

This book presents the theory and applications of multirate filtering with the extensive use of MATLAB including the *Signal Processing, Filter Design, and Wavelet Toolboxes*. The material in the text is supported by examples solved in MATLAB aimed to provide experiments that illustrate and verify the underlying theory. The solved MATLAB examples given through the book and the MATLAB exercises given at the end of each chapter enable the reader to develop deeper understanding of the multirate filtering problems.

The benefit of this book is a convenient access to the theory, design and implementation of multirate filters.

The book is divided in 12 chapters.

Chapter I presents the background review of the single-rate discrete-time signals and systems. A concise review of the time-domain and the transform-domain characterization of discrete-time signals and systems is given. First, we discuss the representation of a discrete-time signal as a sequence of numbers, and explain the operations on sequences. Then, the definition and properties of discrete-time systems are given with the emphasis to the linear-time-invariant (LTI) systems. The representation in the transform domain comprises the discrete-time Fourier transform (DTFT), the discrete Fourier Transform (DFT), and the z-transform. The definitions of the discrete-time system transfer function and the frequency responses are given. The basic realization structures for FIR and IIR systems are briefly described. Finally, the relations between continuous-time and discrete-time signals are given.

Chapter II is devoted to the basics of multirate systems. This chapter considers the basic sampling rate alterations when changing the sampling rate by an integer factor. The time-domain representations of down-sampling and up-sampling operations are introduced with the emphasis to the linearity and time-dependence properties. The z-domain and frequency-domain representations of down-sampled and up-sampled signals are developed. The spectrum of the down-sampled signal is analyzed, and the concept of aliased spectra is introduced. The spectrum of the up-sampled signal has been analysed too, and the appearance of images in the signal spectrum is explained. At this point, the essential importance of filtering has been observed. The concept of decimation and interpolation that include filtering as an integral part of a sampling rate alteration operation has been explained next. The description of Six Identities that enable the reductions in computational complexity of multirate systems is given. Then, the effects of the sampling rate conversion with the phase offset are described. The polyphase decomposition of the sequence and the representation of polyphase components are explained in detail. Finally, the concept of multistage multirate system is presented.

Chapter III considers the general role of filters in multirate systems. The spectral characteristics of decimators and interpolators are discussed first. The effects of aliasing and imaging are illustrated by means of examples. Following the discussion on aliasing and imaging, the problem of proper filter specifications that could ensure the suppression of the aliased spectra and the removal of images has been underlined. Three commonly used types of filter specifications are described. In the sequel, it is shown by means of numerous examples how the existing MATLAB functions for FIR and IIR filter design can be used to meet the typical specifications. The special attention has been focussed to the computation of the residual aliasing, which is inevitably left after filtering. It is shown how the aliasing characteristics of the decimation filter can be computed. The sampling rate alteration of bandpass signals is also discussed in this chapter.

The design and implementation of FIR filters for sampling rate conversion is presented in **Chapter IV**. The implementation structures of decimators and interpolators that are based on FIR filtering are considered in this chapter. First, the application of the FIR filter direct implementation forms in constructing decimators and interpolators are analyzed. The central part of the chapter is devoted to the description of the efficient polyphase implementation of decimators and interpolators. The use of MATLAB for the verification and simulation of the decimator/interpolator structure is demonstrated. The operation of those structures is illustrated by means of example decimators and interpolators. Also, the polyphase memory-saving structures for decimators and interpolators are shown. In this chapter, the computational efficiency of FIR decimators and interpolators is discussed in order to demonstrate the significant computational savings achieved in FIR multirate filtering.

Chapter V is devoted to IIR filters for sampling rate conversion. In this chapter, the direct implementation structure for IIR decimators and interpolators has been considered first. The computational efficiency of IIR decimators and interpolators when implemented in the direct form has been presented. The application of the polyphase decomposition in constructing efficient IIR decimators and interpolators has been considered. The advantage of the solutions based on all-pass polyphase components has been underlined and illustrated by means of an example. The role of extra filter in constructing high-performance IIR decimator and interpolator is explained and illustrated. In this chapter, the particular attention has been paid to the solutions which use the implementation structures based on the parallel connection of two all-pass subfilters. It is shown that extremely efficient IIR decimators and interpolators can be achieved when using the cascade of halfband IIR filters followed by the factor-of-two down-samplers. The application of elliptic minimal Q factors (EMQF) filters in the systems for sampling rate alterations has been shown.

Chapter VI considers the sampling rate conversion by a fractional factor, sometimes called a fractional sampling rate conversion. It is shown first how the MATLAB functions can be used to convert the sampling rate of the signal by a rational factor. The technique for constructing efficient sampling rate conversion by a rational factor based on FIR filters and polyphase decomposition is presented. In the sequel, we consider the sampling rate alteration with an arbitrary conversion factor. We present the polynomial-based approximation of the impulse response of a hybrid analog/digital model, and the implementation based on the Farrow structure. We also consider the construction of fractional delay filters. MATLAB examples illustrate the applications.

Chapter VII is devoted to the theory and design of Lth-band filters and particularly to the halfband filters, the most important subclass of Lth-band filters. This chapter starts with the linear-phase Lth-band FIR filters. We introduce the main definitions and present by means of examples the efficient polyphase implementation of Lth-band FIR filters. We discuss the properties of the separable linear-phase transfer functions, and construct the minimum-phase and maximum-phase FIR transfer functions. The minimum-phase (maximum-phase) transfer function is considered as a spectral factor of the separable (factorisable)

FIR filter transfer function. In sequel, we present the design and efficient implementation of the halfband FIR filters. A halfband filter can be considered as a special class of the Lth-band filter obtained for L = 2. The class of IIR Lth-band and halfband filters is presented next. The particular attention is addressed to the design and implementation of IIR halfband filters.

In **Chapter VIII** we present the complementary filter pairs. First, we review the definitions of delay-complementary, all-pass complementary, power-complementary and magnitude complementary properties. The generation of a highpass filter (FIR and IIR) from the complementary lowpass filter is shown. Then, the definitions of the analysis and synthesis lowpass/highpass filter pairs are given. In the sequel, we present the design and implementation of FIR filter pairs comprising: delay-complementary, power-complementary, and magnitude complementary FIR filter pairs. The design and implementation of three classes of IIR filter pairs satisfying the allpass-complementary/power-complementary, power-complementary, and allpass-complementary/magnitude-complementary properties are presented in this chapter. We demonstrate the high-performance complementary IIR filter pairs, which benefit the advantages of FIR and IIR filter properties.

In **Chapter IX** we present the application of multirate techniques in filter design and implementation. The chapter considers filters with equal input and output sampling rate, with narrow transition bandwidths that are very difficult to be implemented by using classical single-rate techniques. Employing the multirate techniques with multistage filtering and the complementary filter pairs, one achieves to construct the overall high-order filter by combining several low-order subfilters. In this way, the overall filtering task is shared between subfilters of significantly lower order. In this chapter, we consider the application of multistage filtering to design the narrowband filters. Extremely efficient solutions are achieved when using halfband decimation and interpolation subfilters. The wideband filters with sharp transition bands are considered, as well. The solutions are based on the complementary multirate filtering and multistage design.

Chapter X considers the applications of frequency-response masking techniques in constructing digital filters with sharp transition bands. The concept of model and masking filters is introduced and the design and implementation of narrowband FIR and IIR filters is discussed. In the sequel, the frequency-response masking approach in designing filters with the arbitrary bandwidths is considered. The concept of frequency-response masking technique based on the model complementary filter pair and two masking filters that is suitable for synthesizing the arbitrary bandwidth filters is presented. The general characteristics of the model complementary filter pair and that of two masking filters are shown. The synthesis of FIR and IIR wideband filters with sharp transition bands is illustrated by means of examples. A solution that uses the halfband filter as one of the masking filters is also given in this chapter.

Chapter XI is devoted to the design and realization of the comb-based filters for decimators and interpolators. In this chapter, we first introduce the concept of the basic cascade integrator-comb (CIC) filter and discuss its properties. Then, we present the structures of the CIC-based decimators and interpolators, discuss the corresponding frequency responses, and demonstrate the overall two-stage decimator constructed as the cascade of a CIC decimator and an FIR decimator. In the next section, we expose the application of the polyphase implementation structure, which is aimed to reduce the power dissipation in the comb-based decimators and interpolators. We consider techniques for sharpening the original comb filter magnitude response and emphasize an approach that modifies the filter transfer function in a manner to provide a sharpened filter operating at the lowest possible sampling rate. Finally, we give a brief description of the modified comb filter based on the zero-rotation approach. We discuss the improvements achieved with modified comb-filter transfer function and sharpening techniques.

The final chapter, **Chapter XII**, illustrates by means of examples the applications of multirate filters in constructing multirate filter banks. First, we give a brief review of the properties of the two-channel analysis and synthesis filter banks with the condition for elimination of aliasing. The perfect-reconstruction and nearly perfect-reconstruction properties are discussed, and solutions based on FIR and IIR QMF banks and the orthogonal two-channel filter banks are shown. In the sequel, the tree-structured multichannel filter banks are considered including the uniform filter banks and nonuniform filter banks with the special emphasis to the octave filter banks. The process of signal decomposition and reconstruction is illustrated by means of examples. The application of some MATLAB functions for signal decomposition and reconstruction (from the *Wavelet Toolbox*) that are based on the octave filter banks has been also demonstrated in this Chapter.

Finally, at the end of each chapter, except Chapter I, numerous MATLAB exercises are provided, with the intention to help the reader in developing various individual solutions. Some of the exercises require only the modifications of the existing programs given in the text. However, some of the exercises are more demanding.

The material exposed in this book range in difficulty from very simple applications of multirate techniques and multirate filtering to more elaborate and demanding multirate processing algorithms.

The MATLAB examples are extensively used through the chapters. In the first chapters, the script files in the form of demo programs are given in details. Later on, the MATLAB applications are shown with the essential code fragments only. Using the given code fragments, the reader can easily complete his/her own m-file and generate the computations and figures of interest.

The majority of examples use the existing MATLAB functions from the *Signal Processing* and *Filter Design Toolboxes* in order to exploit the power of MATLAB for the easier access to the main subject of this book. In the last chapter, some functions from the *Wavelet Toolbox* are utilized.

Although the MATLAB programs in this book are written in a simple intuitive way, it is expected that the reader possess some basic knowledge in MATLAB programming.

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