Fractal Top for Contractive and Non-Contractive Transformations

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ABSTRACT

Barnsley (2006) introduced the notion of a fractal top, which is an addressing function for the set attractor of an Iterated Function System (IFS). A fractal top is analogous to a set attractor as it is the fixed point of a contractive transformation. However, the definition of IFS is extended so that it works on the colour component as well as the spatial part of a picture. They can be used to colour-render pictures produced by fractal top and stealing colours from a natural picture. Barnsley has used the one-step feedback process to compute the fractal top. In this paper, the authors introduce a two-step feedback process to compute fractal top for contractive and non-contractive transformations.

Keywords: Colour Stealing, Contractive Transformation, Fixed Point, Fractal Top, Hénon Transformation, Iterated Function System (IFS), Non-Contractive Transformation, Non-Expansive Transformation

1. INTRODUCTION

Points in a space, such as the real interval [0, 1] and the euclidean plane $\mathbb{R}^2$, are organized by means of a system of addresses or coordinates. For example, points in $\mathbb{R}^2$ may be addressed by ordered pairs of decimal expansions. Addresses are themselves members of certain types of spaces called code spaces (see, for instance, Devaney (1986, 1992), Jacquin (1992) and Parry (1966)).

A code space is a significant set, which consists of uncountable infinite points. Its importance lies in the fact that it can be embedded in a very small interval. The power of code space in the study of fractals is significant. It is used to give coordinates to the points of a self-similar set. The associated theory has

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potential applications in biological sciences, relating the biology and human anatomy with the topology and code space in mathematical sciences. For details regarding code space and its association with fractal generation, one may refer to Devaney (1986, 1992), Jacquin (1992) and Parry (1966).


The study of fractal top is initiated by Barnsley (2005, 2006). It is an addressing function for a set attractor of an IFS, which assigns a unique largest address to each point of the set attractor. Until now, set and measure attractors were the only known fixed points of an IFS. However, it has recently been shown in Barnsley (2006, p. 352) that fractal top is the unique largest fixed point of a contractive transformation associated with certain code space.

Fractal top may be obtained by deterministic algorithms or by a variant of chaos game. This new operator not only works on the pixel co-ordinates of the picture but on the colour component as well. It defines a picture that is invariant under an IFS consisting of contractive transformations which appears to have theoretical as well as practical importance in mathematical sciences and simulation of natural objects.

Colour stealing is an emerging area of computer graphics with a wide range of applications. Natural colouring is a complex phenomenon to simulate the objects on the computer. For example, natural colour of a leaf from a tree varies in time and space both as sunlight, temperature and other weather conditions affect the colour of the leaf. Thus it is difficult to simulate the colour of the leaf on a computer. Colour stealing extracts natural colours from images and renders the set attractor with true natural colours. The process of colour stealing may be considered as a variant of texture map in which the objective of the map is to map the texture of real world image to the synthetic image generated by an IFS. It has potential applications in computer graphics, image processing and image compression. One of the important applications of colour stealing is to render synthetic pictures in true natural colours taken from any digitized image with the same intensity, depth and width of the pixel. Barnsley (2006) extended the concept of colour stealing and rendered the fractal top by taking colours from a real world image. He described this process as tops plus colour stealing. A picture produced by tops plus colour stealing carries a natural palette, possesses continuity and discontinuity along with some kind of self-similarity. However, a fractal top may also be defined in terms of lifted IFS so that a chaos game may suitably be applied, so as to take care of an additional component of address. The formulation of lifted IFS (given in Preliminaries) is useful for keeping track of the highest address encountered along random orbits.

Barnsley has obtained fractal top using function iterative procedure for a contractive IFS. However, we, in this paper, give an algorithm to calculate the fractal top using superior iterative procedure for a non-contractive IFS and include some examples to visualize definitive effect of colour stealing. We extend the definition of lifted IFS in terms of a non-expansive transformation.

We also take an example that involves much more complicated transformation, namely the Hénon transformation and obtain the fractal top of this transformation. The Hénon map introduced by Michel Hénon (1976) is chaotic in nature and has an attractor at some canonical values which is smooth in one direction and a Cantor set in another. For theory and applications related to Hénon attractor, one may refer
Load Balancing for the Dynamic Distributed Double Guided Genetic Algorithm for MAX-CSPs
www.igi-global.com/article/load-balancing-dynamic-distributed-double/49684?camid=4v1a