Functional Networks

Oscar Fontenla-Romero
University of A Coruña, Spain

Bertha Guijarro-Berdiñas
University of A Coruña, Spain

Beatriz Pérez-Sánchez
University of A Coruña, Spain

INTRODUCTION

Functional networks are a generalization of neural networks, which is achieved by using multiargument and learnable functions, i.e., in these networks the transfer functions associated with neurons are not fixed but learned from data. In addition, there is no need to include parameters to weigh links among neurons since their effect is subsumed by the neural functions. Another distinctive characteristic of these models is that the specification of the initial topology for a functional network could be based on the features of the problem we are facing. Therefore knowledge about the problem can guide the development of a network structure, although on the absence of this knowledge always a general model can be used.

In this article we present a review of the field of functional networks, which will be illustrated with practical examples.

BACKGROUND

Artificial Neural Networks (ANN) are a powerful tool to build systems able to learn and adapt to their environment, and they have been successfully applied in many fields. Their learning process consists of adjusting the values of their parameters, i.e., the weights connecting the network’s neurons. This adaptation is carried out through a learning algorithm that tries to adjust some training data representing the problem to be learnt. This algorithm is guided by the minimization of some error function that measures how well the ANN is adjusting the training data (Bishop, 1995). This process is called parametric learning. One of the most popular neural network models are Multilayer Perceptrons (MLP) for which many learning algorithms can be used: from the brilliant backpropagation (Rumelhart, Hinton & Williams, 1986) to the more complex and efficient Scale Conjugate Gradient (Möller, 1993) or Levenberg-Marquardt algorithms (Hagan & Menhaj, 1994).

In addition, also the topology of the network (number of layers, neurons, connections, activation functions, etc.) has to be determined. This is called structural learning and it is carried out mostly by trial and error.

As a result, there are two main drawbacks in dealing with neural networks:

1. The resulting function lacks of the possibility of a physical or engineering interpretation. In this sense, Neural Networks act as black boxes.

2. There is no guarantee that the weights provided by the learning algorithm correspond to a global optimum of the error function, it can be a local one.

Models like Generalized Linear Networks (GLN) present an unique global optimum that can be obtained by solving a set of linear equations. However, its mapping function is limited as this model consists of a single layer of adaptive weights ($w_j$) to produce a linear combination of non linear functions ($\phi_j$):

$$y(x) = \sum_{j=0}^{M} w_j \phi_j(x).$$

Some other popular models are Radial Basis Function Networks (RBF) whose hidden units use distances to a prototype vector ($\mu_j$) followed by a transformation with a localized function like the Gaussian:
Functional Networks

\[ y(x) = \sum_{j=0}^{M} w_j \exp \left( -\frac{\|x - j\|^2}{2\sigma_j^2} \right) \]

The resulting architecture is more simple than the one of the MLP, therefore reducing the complexity of structural learning and propitiating the possibility of physical interpretation. However, they present some other limitations like their inability to distinguish non significant input variables (Bishop, 1995), to learn some logic transformations (Moody & Darken, 1989) or the need of a large number of nodes even for a linear map if precision requirement is high (Youssef, 1993).

Due to these limitations, there have been appearing some models that extend the original ANN, such as, fuzzy neural networks (Gupta & Rao, 1994), growing neural networks, or probabilistic neural networks (Specht, 1990). Nowadays, the majority of these models still act as black boxes. Functional networks (Castillo, 1998, Castillo, Cobo, Gutiérrez, & Pruneda, 1998), a relatively new extension of neural networks, take into account the functional structure and properties of the process being modeled, that naturally determine the initial network’s structure. Moreover, the estimation of the network’s weights it is often based on an error function that can be minimized by solving a system of linear equations, therefore conducting faster to an unique and global solution.

**DESCRIPTION OF FUNCTIONAL NETWORKS**

Functional networks (FN) are a generalization of neural networks, which is achieved by using multiargument and learnable functions (Castillo, 1998, Castillo, Cobo, Gutiérrez, & Pruneda, 1998), i.e., the shape of the functions associated with neurons are not fixed but learned from data. In this case, it is not necessary to include weights to ponder links among neurons since their effect is subsumed by the neural functions. Figure 1 shows an example of a general FN for \( I=N \) explanatory variables.

Functional networks consist of the following elements:

- Several layers of storing units (represented in Figure 1 by small filled circles). These units are used for the storage of both the input and the output of the network, or to storage intermediate information (see units \( y_{(k)} \) in Figure 1).
- One or more layers of functional units or neurons (represented by open circles with the name of each of the functional units inside). These neurons include a function that can be multivariate and that can have as many arguments as inputs. These arguments, and therefore the form of the neural functions, are learnt during training. By applying their functions, neurons evaluate a set of input parameters.