Hierarchical Neuro–Fuzzy Systems Part I

Marley Vellasco
PUC-Rio, Brazil

Marco Pacheco
PUC-Rio, Brazil

Karla Figueiredo
UERJ, Brazil

Flavio Souza
UERJ, Brazil

INTRODUCTION

Neuro-fuzzy [Jang,1997][Abraham,2005] are hybrid systems that combine the learning capacity of neural nets [Haykin,1999] with the linguistic interpretation of fuzzy inference systems [Ross,2004]. These systems have been evaluated quite intensively in machine learning tasks. This is mainly due to a number of factors: the applicability of learning algorithms developed for neural nets; the possibility of promoting implicit and explicit knowledge integration; and the possibility of extracting knowledge in the form of fuzzy rules. Most of the well known neuro-fuzzy systems, however, present limitations regarding the number of inputs allowed or the limited (or nonexistent) form to create their own structure and rules [Nauck,1997][Nauck,1998][Vuorimaa,1994][Zhang,1995].

This paper describes a new class of neuro-fuzzy models, called Hierarchical Neuro-Fuzzy BSP Systems (HNFBS). These models employ the BSP partitioning (Binary Space Partitioning) of the input space [Chrysanthou,1996] and have been developed to bypass traditional drawbacks of neuro-fuzzy systems. This paper introduces the HNFB models based on supervised learning algorithm. These models were evaluated in many benchmark applications related to classification and time-series forecasting. A second paper, entitled Hierarchical Neuro-Fuzzy Systems Part II, focuses on hierarchical neuro-fuzzy models based on reinforcement learning algorithms.

BACKGROUND

Hybrid Intelligent Systems conceived by using techniques such as Fuzzy Logic and Neural Networks have been applied in areas where traditional approaches were unable to provide satisfactory solutions. Many researchers have attempted to integrate these two techniques by generating hybrid models that associate their advantages and minimize their limitations and deficiencies. With this objective, hybrid neuro-fuzzy systems [Jang,1997][Abraham,2005] have been created.

Traditional neuro-fuzzy models, such as ANFIS [Jang,1997], NEFCLASS [Nauck,1997] and FSOM [Vuorimaa,1994], have a limited capacity for creating their own structure and rules [Souza,2002a]. Additionally, most of these models employ grid partition of the input space, which, due to the rule explosion problem, are more adequate for applications with a smaller number of inputs. When a greater number of input variables are necessary, the system’s performance deteriorates.

Thus, Hierarchical Neuro-Fuzzy Systems have been devised to overcome these basic limitations. Different models of this class of neuro-fuzzy systems have been developed, based on supervised technique.

HIERARCHICAL NEURO-FUZZY SYSTEMS

This section presents the new class of neuro-fuzzy systems that are based on hierarchical partitioning.
Two sub-sets of hierarchical neuro-fuzzy systems (HNF) have been developed, according to the learning process used: 

1. Supervised learning models (HNFB [So uza, 2002b], [Vellasco, 2004], HNFB [Gonçalves, 2006]).
2. RL-HNFB (Figueiredo, 2005a).
3. RL-HNFB-Mamdani (Bezerra, 2005).

The focus of this paper is on the first sub-set of models, which are described in the following sections.

**HIERARCHICAL NEURO-FUZZY BSP MODEL**

**Basic Neuro-Fuzzy BSP Cell**

An HNFB cell is a neuro-fuzzy mini-system that performs fuzzy binary partitioning of the input space. The HNFB cell generates a crisp output after a defuzzification process.

Figure 1(a) illustrates the cell’s functionality, where ‘\(x\)’ represents the input variable; \(\rho(x)\) and \(\mu(x)\) are the membership functions low and high, respectively, which generate the antecedents of the two fuzzy rules; and \(y\) is the crisp output. The linguistic interpretation of the mapping implemented by the HNFB cell is given by the following rules:

- If \(x \in \rho\) then \(y = d_1^i\)
- If \(x \in \mu\) then \(y = d_2^i\).

Each rule corresponds to one of the two partitions generated by BSP. Each partition can in turn be subdivided into two parts by means of another HNFB cell.

The profiles of membership functions \(\rho(x)\) and \(\mu(x)\) are complementary logistic functions.

The output \(y\) of an HNFB cell (defuzzification process) is given by the weighted average. Due to the fact that the membership function \(\rho(x)\) is the complement to 1 of the membership function \(\mu(x)\), the following equation applies:

\[
y = \rho(x) \cdot d_1^i + \mu(x) \cdot d_2^i \quad \text{or} \quad y = \sum_{i=1}^{\hat{\alpha}} \alpha_i \cdot d_i
\]

where \(\alpha_i\) symbolizes the firing level of the rule in partition \(i\) and are given by: \(\alpha_i = \rho(x)\); \(\alpha_i = \mu(x)\). Each \(d_i\) corresponds to one of the three possible consequents below:

- A singleton: The case where \(d_i = \text{constant}\).
- A linear combination of the inputs:

\[
d_i = \sum_{k=1}^{d} w_k x_k + w_0
\]

where: \(x_k\) is the system’s \(k\)-th input; the \(w_k\) represent the weight associated with the input \(x_k\); \(‘n’\) is equal to the total number of inputs; and \(w_0\) corresponds to a constant value.

- The output of a stage of a previous level: The case where \(d_i = y_p\), where \(y\) represents the output of a generic cell ‘\(j\)’, whose value is also calculated by eq. (1).

**HNFB Architecture**

An HNFB model may be described as a system that is made up of interconnections of HNFB cells. Figure 1(b) illustrates an HNFB system along with the respective partitioning of the input space. In this system, the initial partitions 1 and 2 (‘BSP0’ cell) have been subdivided; hence, the consequents of its rules are the outputs of BSP1 and BSP2, respectively. In turn, these subsystems have, as consequents, values \(y_{11}, y_{12}, y_{21}\) and \(y_{22}\), respectively. Consequent \(y_{ij}\) is the output of the ‘BSP12’ cell. The output of the system in figure 1(b) is given by equation (2).

\[
y = \alpha_1 (\alpha_{11} d_{11} + \alpha_{12} (\alpha_{21} d_{21} + \alpha_{22} d_{22})) + \alpha_2 (\alpha_{21} d_{21} + \alpha_{22} d_{22})
\]

\(\text{(2)}\)

It must be stressed that, although each BSP cell divides the input space only in two fuzzy set (low and high), the complete HNFB architecture divides the universe of discourse of each variable in as many partitions as necessary. The number of partitions is determined during the learning process. In Figure 1(c), for instance, the upper left part of the input space (partition 12 in gray) has been further subdivided by the horizontal variable \(x_j\), resulting in three fuzzy sets for the complete universe of discourse of this specific variable.

**Learning Algorithm**

The HNFB system has a training algorithm based on the gradient descent method for learning the structure.
Related Content

Learning Bayesian Networks
www.igi-global.com/chapter/learning-bayesian-networks/24286?camid=4v1a

Collusion-Free Privacy Preserving Data Mining
www.igi-global.com/article/collusion-free-privacy-preserving-data/46962?camid=4v1a

Combining Requirements Engineering and Agents
www.igi-global.com/chapter/combining-requirements-engineering-agents/24289?camid=4v1a

Incremental Load in a Data Warehousing Environment
www.igi-global.com/article/incremental-load-data-warehousing-environment/45153?camid=4v1a