Linear Programming

Xiaofeng Zhao
University of Mary Washington, USA

INTRODUCTION

Linear programming (LP or linear optimization) deals with the problem of the optimization (minimization or maximization), in which a linear objective function is optimized subject to a set of linear constraints. Its name means that planning (programming) is being done with a mathematical model. It is one of widely used techniques in operations research and management science. Some typical applications are: 1. a manufacture wants to develop a production schedule and an inventory policy that will satisfy sales demand in future periods. Ideally, the schedule and policy will enable the company to satisfy demand and at the same time minimize the total production and inventory costs. 2. A financial analyst must select an investment portfolio from a variety of stock and bond investment alternatives. The analyst would like to establish the portfolio that maximizes the return on investment. 3. A marketing manager wants to determine how best to allocate a fixed advertising budget among alternative advertising media such as radio, television, newspaper, and magazine. The manager would like to determine the media mix that maximizes the advertising effectiveness. 4. A company has warehouse in a number of locations throughout a country. For a set of customer demands, the company would like to determine how much each warehouse should ship to each customer so that total transport costs are minimized. These examples are only a few of situations in which linear programming has been used successfully, but illustrate the diversity of linear programming applications (Anderson et al., 2011). Other applications include supply chain management in the motor industry, productions scheduling in the brewing industry, aircraft crew and production scheduling, financial planning and capital budgeting, asset and liability management, energy management in the utilities sector, network design in the telecommunications sector and transportation (Dowman & Wilson, 2002).

Linear programming, a specific case of mathematical programming, has some properties in common. Linear programs can be expressed in canonical form:

Maximize $c^T x$

Subject $Ax \leq b$

and $x \geq 0$

We are given an $m$-vector, $b = (b_1, \ldots, b_m)$, an $n$-vector, $c = (c_1, \ldots, c_m)$, and an $m \times n$ matrix $A$, where $x$ represents the vector of decision variables (to be determined), $c$ and $b$ are vectors of coefficients, $A$ is a matrix of coefficients, and $(\cdot)^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The inequality $Ax \leq b$ is the constraint which specifies a convex polytope over which the objective function is to be optimized. The column vector $b$ which is referred to as the right-hand-side vector represents the maximal requirements to be satisfied. A set of variables $x$ satisfying all the constraints is called feasible point or a feasible vector. The set of all such points constitutes the feasible region. Using the foregoing terminology, the linear programming problem can be stated as follows: among all feasible regions, find one that minimizes (or maximize) the objective function. It is fairly common for large linear programming models to in-
include a mixture of functional constraints, some
with “≤” signs, some with “≥” signs, and some
with “=” signs.

Many people have contributed to the growth
of linear programming by developing its math-
ematical theory, devising efficient computational
methods and codes, exploring new algorithms
and new applications, and by their use of linear
programming as an aiding tool for solving more
complex problems, for instance, discrete programs,
nonlinear programs, combinatorial problems,
stochastic programming problems and problems
of optimal control.

This article first presents the background of
linear programming and then focuses on concepts,
characteristics, techniques, general theories,
and effective solution algorithms of linear pro-
gramming optimization problems. The simplex
algorithm provides considerable insights into
the theory of linear programming and yields an
efficient algorithm in practice. It also presents
Karmarkar’s polynomial-time algorithm for linear
programming problems because it compares fa-
vorably with the simplex method, particularly for
general large-scale problems. At last, it indicates
future trends and makes a conclusion.

BACKGROUND

Linear programming problem was first conceived
by George B. Danzig around 1947 while he was
first mathematical advisor to the United States Air
Force Comptroller on developing a mechanized
planning tool for a time-staged development,
training, and logistical supply program. Soviet
mathematician and economist Leonid Kantorovich
developed the earliest linear programming prob-
lems in 1939 for use during World War II to plan
expenditures and returns in order to reduce costs
to the army and increase losses to the enemy. But
his work remains unknown until 1959. Hence the
conception of the general classes of linear pro-
gramming problem is usually credited to Danzig.
Because the Air Force refers to its various plans
and schedules to be implemented as “programs,”
the method was kept secret until 1947 when George
B. Dantzig published the simplex method and John
von Neumann developed the theory of duality as
a linear optimization solution, and applied it in
the field of game theory. Danzig’s first published
paper addressed this problem as “Programming
in a Linear Structure.” The term linear program-
ing was actually coined by the economist and
mathematician T.C. Koopmans in the summer of
1948 while he and Danzig strolled near the Santa
Monica beach in California (Bazaraa et al., 2009).

Due to the wide applicability of linear pro-
gramming models, an immense amount of work
has appeared regarding theory and algorithms
for LP. During and after World War II it became
evident that planning and coordinating among
various projects and the efficient utilization of
scarce resources were essential. Interest in linear
programming spread quickly among industrial
engineering, management, operations research,
computer science, economics, mathematicians,
statistics, and government institutions.

Linear programming is a considerable field of
optimization for several reasons. Many practical
problems in operations research and management
science can be expressed as linear programming
problems. Certain special cases of linear pro-
gramming, such as network flow problems and
multi-commodity flow problems are considered
important enough to have generated much research
on specialized algorithms for their solution. His-
torically, ideas from linear programming have
inspired many of the central concepts of optimiza-
tion theory, such as duality, decomposition, and the
importance of convexity and its generalizations.
Although the modern management issues are ever
changing, most companies would like to maximize
profits or minimize costs with limited resources.
Therefore, many issues can be characterized as
linear programming problems. It is not surprising
that in a recent survey of Fortune 500 companies,
85% of those responding said that they had used
linear programming (Winston & Albright, 2012).
The history, theory, and applications of linear
Related Content

Loss Profit Estimation Using Temporal Association Rule Mining
[www.igi-global.com/article/loss-profit-estimation-using-temporal-association-rule-mining/142780?camid=4v1a](www.igi-global.com/article/loss-profit-estimation-using-temporal-association-rule-mining/142780?camid=4v1a)

A Method for Scalable Real-Time Network Performance Baselining, Anomaly Detection, and Forecasting
[www.igi-global.com/article/method-scalable-real-time-network/65536?camid=4v1a](www.igi-global.com/article/method-scalable-real-time-network/65536?camid=4v1a)

Global Optimization of Economic and Social Policy
[www.igi-global.com/chapter/global-optimization-of-economic-and-social-policy/107307?camid=4v1a](www.igi-global.com/chapter/global-optimization-of-economic-and-social-policy/107307?camid=4v1a)

Parallelization and Load Balancing Techniques for HPC
[www.igi-global.com/chapter/parallelization-and-load-balancing-techniques-for-hpc/107367?camid=4v1a](www.igi-global.com/chapter/parallelization-and-load-balancing-techniques-for-hpc/107367?camid=4v1a)