TOPSIS in Business Analytics

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INTRODUCTION

Multi-attribute decision making (MADM) and multi-criteria decision making (MCDM) problems with \( m \) alternatives that are evaluated by \( n \) attributes may be viewed as a geometric system with \( m \) points in \( n \)-dimensional space. Hwang and Yoon (1981) developed the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) based on the concept that the chosen alternative should have the shortest distance from the positive-ideal solution (PIS) and the longest distance from the negative-ideal solution (NIS). This principle has been also suggested by Zeleny (1982) and Hall (1989), and it has been enriched by Yoon (1987) and Hwang, Lai, and Liu (1993). Further discussion was made by many (Chu, 2002; Olson, 2004; Peng, 2000). The PIS has the best measures over all attributes, while the NIS has the worst measures over all attributes (Wu, 2006). An ideal solution is defined as a collection of ideal levels (or ratings) in all attributes considered. It is assumed that the true ideal solution is usually unattainable or infeasible so to be as close as possible to such an ideal solution is the rationale of human choice. TOPSIS is one of the most popular MCDM methods (Ozturk, 2011).

In this chapter we describe the methodology for TOPSIS, provide a few examples in decision making each to illustrate TOPSIS, briefly mention the role of technology that might be used in obtaining solution, and the interpretation of the TOPSIS solution. We present some the strengths and weaknesses to the process.

BACKGROUND

TOPSIS was the result of work done by Yoon and Hwang (1980). TOPSIS has been used in a wide spectrum of comparisons of alternatives including: item selection from among alternatives, ranking leaders or entities, remote sensing in regions, data mining, and supply chain operations. TOPSIS is chosen over other methods because it orders the feasible alternatives according to their closeness to an ideal solution (Malezewski, 1996).

Napier (1992) provided some analysis of the use of TOPSIS for the department of defense in industrial base planning and item selection. For years the military used TOPSIS to rank order the systems’ request from all the branches within the service for the annual budget review process (Fox, 2012) as well as being taught again in as part of decision analysis. Current work is being done to show the ability of TOPSIS to rank order nodes of a dark or social network across all the metrics of social network analysis (Fox, 2012; Fox & Everton, 2013).

In manufacturing analysis, Wang et al. (2008) proposed two methods to improve TOPSIS for multi-response optimization using Taguchi’s loss function. Ozturk and Batuk (2011) used TOPSIS for spatial decisions and then linked to geographical information systems (GIS) operations for flood vulnerability. Olson and Wu (2005, 2006) have shown how TOPSIS may be used for data mining and analysis in credit card score data. Olson (2006) presented a comparison of weights (centroid weights, equal weights, and weights by linear regression) in TOPSIS models using baseball data where their conclusion is that accurate weights in TOPSIS are crucial to success.
In a business setting it has been applied to a large number of application cases in advanced manufacturing processes (Agrawal, Kohli, & Gupta, 1991; Parkan & Wu, 1999), purchasing and outsourcing (Kahraman, Engin, Kabak, & Kaya, 2009; Shyura & Shih, 2006), and financial performance measurement (Feng & Wang, 2001).

In social networks, TOPSIS has been used to rank order the nodes across all metrics in order to identify the most influential node (Fox, et al. 2013).

**MAIN FOCUS OF THE CHAPTER: TOPSIS**

**TOPSIS Methodology**

The TOPSIS process is carried out as follows:

**Step 1:** Create an evaluation matrix consisting of $m$ alternatives and $n$ criteria, with the intersection of each alternative and criteria given as $x_{ij}$, giving us a matrix $(X_{ij})_{m \times n}$.

$$
D = 
\begin{bmatrix}
    x_1 & x_2 & x_3 & \ldots & x_n \\
x_{11} & x_{12} & x_{13} & \ldots & x_{1n} \\
x_{21} & x_{22} & x_{23} & \ldots & x_{2n} \\
x_{31} & x_{32} & x_{33} & \ldots & x_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & x_{m3} & \ldots & x_{mn}
\end{bmatrix}
$$

**Step 2:** The matrix shown as $D$ above then normalized to form the matrix $R=(R_{ij})_{m \times n}$ using the normalization method

$$
r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

for $i=1,2\ldots,m$; $j=1,2\ldots,n$

**Step 3:** Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

**Step 3a:** Use either the decision maker’s weights for the attributes $x_{ij}$, or compute the weights through the use Saaty’s (1980) AHP’s decision maker weights method to obtain the weights as the eigenvector to the attributes versus attribute pair-wise comparison matrix.

$$
\sum_{j=1}^{n} w_j = 1
$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

**Step 3b:** Multiply the weights to each of the column entries in the matrix from Step 2 to obtain the matrix, $T$.

$$
T = (t_{ij})_{m \times n} = (w_j r_{ij})_{m \times n}, i = 1,2,\ldots,m
$$

**Step 4:** Determine the worst alternative ($A_w$) and the best alternative ($A_b$): Examine each attribute’s column and select the largest and smallest values appropriately. If the values imply larger is better (profit) then the best alternatives are the largest values and if the values imply smaller is better (such as cost) then the best alternative is the smallest value.

$$
A_w = \left\{ \max_{i} (t_{ij}) | i = 1,2,\ldots,m \right\} \quad \text{for} \quad j \in J,
$$

$$
A_b = \left\{ \min_{i} (t_{ij}) | i = 1,2,\ldots,m \right\} \quad \text{for} \quad j \in J.
$$