Chapter 21
Linguistic Multi-Attribute Decision Making with a Prioritization Relationship

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ABSTRACT
In this paper we consider linguistic information aggregation problems where a prioritization relationship exists over attributes. The authors define a prioritized 2-tuple ordered weighted averaging (PTOWA) operator to aggregate satisfactions of alternatives under attributes with a linear prioritized ordering. The authors then use the PTOWA and a TOWA operator to aggregate linguistic information where attributes are partitioned into some categories of which prioritization between categories exists. Finally, two illustrative examples are employed to show the feasibility of the proposed method.

1. INTRODUCTION
Due to the complexity and uncertainty of the objective world, as well as the fuzziness of the human mind, some attributes are more suitable to be evaluated in the form of language (Herrera & Verdegay, 1993; Herrera & Herrera-Viedma, 1997; Torra, 1997; Herrera & Martínez, 2000; Herrera & Martínez, 2001; Xu, 2007; Wei, Feng & Zhang, 2009). For example, when evaluating the comprehensive qualities of the students or the performance of cars, the decision makers prefer to use “excellent,” “good” and “poor” for judgment. For linguistic information aggregation, various linguistic aggregation operators have been proposed, including linguistic OWA operator (Herrera & Verdegay, 1993), induced-linguistic OWA operator (Herrera & Herrera-Viedma, 1997), linguistic WOWA operator (Torra, 1997), etc. In the aggregation process using these operators, the results do not exactly match any of the initial linguistic terms. Therefore, an approximation

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process is developed to express the results in the initial expression domain, but leads to the loss of information and lack of precision. Herrera and Martínez (2000) presented an analytical method based upon 2-tuple for linguistic aggregation. They proposed both 2-tuple weighted average (TWA) operator and 2-tuple ordered weighted averaging (TOWA) operator (Herrera & Martínez, 2000), and then successfully applied the TOWA operator to multigranular hierarchical linguistic contexts in a multi-expert decision making problem (Herrera & Martínez, 2001). Many achievements have been taken in multi-attribute decision making (MADM) with these linguistic aggregation operators.

It is important to see that the above linguistic aggregation operators have the ability to trade off between attributes. While in some situations where a prioritization relationship over the attributes exists, we do not expect this kind of compensation. Yager (2004) studied this kind of problem where decision information is described by real numbers. He pointed out that the importance weights of lower priority attributes were based on the satisfaction of alternative to the higher priority attributes. Based on this idea, Yager proposed the prioritized average (PA) operator (Yager, 2008) and the prioritized ordered weighted averaging (POWA) operator (Yager, 2009). Later Wei and Tang (2012) proposed two averaging operators, a generalized PA operator and a generalized POWA operator. In the case with one attribute in each priority category, both operators reduce to the PA operator and the POWA operator proposed by Yager.

Motivated by the above-mentioned studies, we consider linguistic aggregation problems where a prioritization relationship exists over the attributes. This paper is structured as follows. In Section 2, we make a brief review of 2-tuple and its related operators. In Section 3, we propose a prioritized 2-tuple ordered weighted averaging (PTOWA) operator and discuss its properties. We then use this operator and a TOWA operator to aggregate satisfactions of attributes by alternatives. The paper is concluded in Section 4.

2. 2-TUPLE LINGUISTIC REPRESENTATION MODEL AND TOWA OPERATORS

For MADM problems with some qualitative attributes, we need to use a linguistic term set to describe the decision information. Herrera and Martínez (2000) introduced a finite and totally ordered discrete linguistic term set: \[ S = \{ s_\alpha | \alpha = 0, 1, \ldots, \tau \} \], whose cardinality value is odd. For example, a set of seven linguistic terms \( s \) could be:

\[
S = \begin{cases} 
  s_0 & \text{extremely poor}, \\
  s_1 & \text{very poor}, \\
  s_2 & \text{poor}, \\
  s_3 & \text{fair}, \\
  s_4 & \text{good}, \\
  s_5 & \text{very good}, \\
  s_6 & \text{extremely good}.
\end{cases}
\]

Furthermore, Herrera defined 2-tuple to aggregate linguistic information:

**Definition 1**: (Herrera & Martínez, 2001) Let \( S = \{ s_0, s_1, \ldots, s_\tau \} \) be a linguistic term set, then the 2-tuple can be obtained by the translation function \( \theta \):

\[
\theta: S \times [-0.5, 0.5] \to S, \quad \theta(s_i) = (s_i, 0), \quad \text{for any } s_i \in S
\]

(1)

**Definition 2**: (Herrera & Martínez, 2001) Let \( S = \{ s_0, s_1, \ldots, s_\tau \} \) be a linguistic term set, \( s_i \in S \) and \( \beta \in [0, \tau] \), a value representing the result of a symbolic aggregation operation, then the 2-tuple can be obtained with the following function:

\[
\Delta: [0, \tau] \to S \times [-0.5, 0.5], \\
\Delta(\beta) = (s_i, \alpha) = \begin{cases} 
  s_i, & i = \text{round}(\beta), \\
  \alpha = \beta - i, & i \in [-0.5, 0.5]
\end{cases}
\]

(2)

where round (\( \ast \)) is the usual round operation.