INTRODUCTION

A typical way to build surface numerical models or Digital Elevation Models (DEMs) for Geographical Information Systems (GIS) is by processing the stereo images obtained from, for example, aerial photography or SPOT satellite data. These GIS can perform many computations involving their geographic databases. The quality control of a geographic database, and in particular the topological and geometric integrity, are, therefore, important topics (Guptill & Morrison, 1995; Harvey, 1997; Laurini & Milleret-Raffort, 1993; Ubeda & Servigne, 1996). The geometric quality control of the stored DEM is what we are concerned with here. “Quality” means the geometric precision measured in terms of the difference between a DEM and a reference DEM (R-DEM). We assume the R-DEM is a faithful model of the actual surface. Its point density may be greater than the DEM point density.

BACKGROUND

In the literature, several unsatisfactory solutions were proposed for the DEM control with respect to a reference. A critical problem in the error estimation (evaluated using the difference referred to in the previous paragraph) is to establish for each selected point of the DEM the corresponding homologous point in the R-DEM. Other kinds of problems and errors are related to the existence of aberrant points, systematization, and so forth. These problems were studied for horizontal errors in maps in Abbas (1994), Grussenmeyer, Hottier, and Abbas (1994), and Hottier (1996a). These authors found that the dissymmetry model-reference was the most important factor to determine homologous pairs.

Several solutions have been proposed for the punctual control method (e.g., recognition algorithms, filtering methods, adjustment of histograms to theoretical laws) without obtaining completely satisfactory results (Dunn, Harrison, & White, 1990; Lester & Chrisman, 1991). Later, Abbas (1994), Grussenmeyer (1994), and Hottier (1996a) presented an alternative to the punctual control method: the linear control method based on the dissymmetry of the Hausdorff distance. Habib (1997) analyzed precision and accuracy in altimetry and mentioned some of the proposals of the last decade for the elevation control of quality.

In the case of the DEMs, to assess the difference that gives rise to the error we wish to compute, we need to identify without ambiguity each point \( M \) in the DEM with its homologous point \( P \) in the R-DEM. Two reasons that make this task difficult follow:

1. Many points \( M \), such as those situated on regular sides, which are indistinguishable from their neighbors, are not identifiable. Potentially identifiable points are those located on sharp slope variations and possibly those with zero slope (tops, bottoms, and passes).
2. A point identifiable on the DEM is not necessarily identifiable in the R-DEM because of a difference in scale (generalization) or aberrant errors.

Finding identifiable homologous pairs of points is difficult for an operator, and automating this is a very delicate process.

In the case of the precision assessment of a DEM from an ASTER (Advanced Spaceborne Thermal Emission and Reflection Radiometer; Hirano, Welch, & Lang, 2002), profiles or benchmarks (particular points) are used for the elevation accuracy assessment. This elevation accuracy assessment refers to the vertical error of the DEM. However, the horizontal error is not taken into account when using profiles and, also, there is a model-reference discrepancy and the choice and number of the benchmarks might not be a good stochastic sample of points.

In light of these difficulties, Zelasco and Ennis (2000) proposed an estimator for the variances of the vertical and horizontal errors. The values obtained for the estimator according to the type of terrain, its unevenness, and the number of sample points in the DEM were studied using...
simulations (Zelasco, 2002; Zelasco, Ennis, & Blangino, 2001). Throughout these studies, it is assumed that the errors in elevation and in the horizontal plane are independent normal random variables—in agreement with Liapounov's theorem (Hottier, 1996b). This proposed method is called the Perpendicular Distance Estimation Method (PDEM).

Let \( e(M_k) = M_k - P_k \) where \( M_k \), \( P_k \) are the \( k \)-selected homologous pairs of points in the DEM and R-DEM, respectively. Estimating \( \sigma(e) \) (variance of the error as distance between \( M \) and \( P \)) without measuring each vector \( e(M_k) \) completely, is the key advantage of the proposed PDEM. Only the projection of each \( e(M_k) \) in particular directions matters. Given an \( M_k \), it becomes unnecessary to find the homologous \( P_k \) in the R-DEM. How this is done is the subject of the following section.

For three-dimensional DEMs built with the usual techniques, such as photogrammetry or remote sensing, there are expected normal deviations in altimetry (\( \sigma_z \)) and in the horizontal plane (\( \sigma_x, \sigma_y \) that are a priori known. They are generally different from each other due to the manner in which the values of the \( x, y \) coordinate (altimetry) and the \( x, y \) coordinates (the horizontal plane) are evaluated. The goal of this PDEM is to evaluate the actual errors (a posteriori) of a given DEM.

**THE QUALITY EVALUATION METHOD**

**Description of the PDEM**

The gap between the DEM and the R-DEM is represented by a function that links each point of the DEM to the vector \( e(M) = M - P \) in \( R^3 \) where \( P \) is the homologous point to \( M \) in the R-DEM. \( M \) and \( P \) represent the same relief element; however, \( P \) is not vertically aligned with \( M \). The DEM precision is a function of the vectors \( e(M_i) \) of a sample of \( \{M_i\} \) points. This function is used in the construction of the covariance matrix \( \sigma(e) \). If the errors in the three directions are different, for instance, in interferometry radar, the rank of \( \sigma(e) \) is three. Its eigenvalues are the variances in the three main coordinates.

When the errors in the orthogonal directions in the plane are equal (stereo images techniques), we can consider that the rank of the covariance matrix \( \sigma(e) \) is two. The eigenvalues are the variances \( \sigma^2_x \) and \( \sigma^2_z = \sigma^2_x = \sigma^2_y \).

In a rotated frame of reference, the new covariance matrix will no longer be diagonal. The new components are expressed in terms of the eigenvalues of \( \sigma \). The following expression corresponds to the first component \( \sigma^2_x \) of the new covariance matrix:

\[
\sigma^2_x = a_{11} \sigma^2_x + a_{12} \sigma^2_y + a_{13} \sigma^2_z
\]  

(1)

where \( a_{ij} \) are the managing cosines of the unit vector in the \( x \) coordinate of the rotated reference frame.

The value of the variance in the \( x \) direction is equal to the square of the distance between the two planes normal to the \( x \) direction: one tangent to the distribution indicative ellipsoid (whose parameters are \( a = \sigma_x; b = \sigma_y; c = \sigma_z \)) and the other passing through its center.

In Figure 1, the plane normal to the \( x \) direction and tangent to the distribution indicative ellipsoid is drawn in the general case.

The algebraic form of equation (1) is used to obtain the variance in any direction \( d \), replacing the coefficients \( a_{ij} \) with the managing cosines \( (\cos \alpha, \cos \beta, \cos \gamma) \) corresponding to this direction \( d \):

\[
\sigma^2_d = \sigma^2_x \cos^2 \alpha + \sigma^2_y \cos^2 \beta + \sigma^2_z \cos^2 \gamma
\]  

(2)

\( \sigma_d \) can be estimated with \( \sum e_i^2/n \) \( 0 < i < 1 \) where the \( e_i \) are the error components in the \( d \) direction.

We construct a mesh of triangles from the points of the R-DEM. We call this a model of the surface. Given a point \( M \) in the DEM, its projection onto the \( x, y \) plane is inside the projection of a unique triangle \( T \) from this model. We call this the corresponding triangle \( T \) to \( M \). The \( e_i \) component in the direction \( d_i \) is the distance \( d_i \) from each point to the plane of the corresponding triangle. The squares of the distances \( d_i \), between each point of the model and the plane of the corresponding triangle, allow us to establish a least squares estimator for the assessment of \( (\sigma^2_x, \sigma^2_y, \sigma^2_z) \).

Recall that if the errors in the horizontal plane are independent of the direction, the problem is reduced from three to two unknowns (see Figure 2). The \( \sigma^2_x = \sigma^2_y \) and the distribution indicative ellipsoid is a revolution ellipsoid. The maximum slope direction in each R-DEM triangle (equal to the angle made by the normal to the triangle and the \( z \) coordinate) will determine the managing cosines direction in two dimensions. The position of the homologous point in the R-DEM does not need to be known. This is the strength of the PDEM. By having determined the corresponding triangle, we capture the required information without needing to find the particular homologous.

![Figure 1. Distribution indicative ellipsoid and tangent plane Π](image-url)