Chapter 9

Decomposition Theorem of Generalized Interval–Valued Intuitionistic Fuzzy Sets

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ABSTRACT

In this chapter, the authors establish decomposition theorems of Generalized Interval-Valued Intuitionistic Fuzzy Sets (GIVIFS) by use of cut sets of generalized interval-valued intuitionistic fuzzy sets. First, new definitions of eight kinds of cut sets generalized interval-valued intuitionistic fuzzy sets are introduced. Second, based on these new cut sets, the decomposition generalized interval-valued intuitionistic fuzzy sets are established. The authors show that each kind of cut sets corresponds to two kinds of decomposition theorems. These results provide a fundamental theory for the research of generalized interval-valued intuitionistic fuzzy sets.

1. INTRODUCTION

In 1965, Zadeh introduced the concept of fuzzy subsets. Latter many authors defined different directions of fuzzy subsets. Atanassov introduced the concept of intuitionistic fuzzy sets (IFSs), which is more generalization of fuzzy subsets and as well as IVFSs. Several authors present a number of results using IFSs. Gargov and Atanassov (1989) introduced the interval-valued intuitionistic fuzzy sets (IVIFs). They have shown several properties on IVIFSs and shown some applications of IVIFSs. Mondal and Samanta (2002) introduced another concept of IFSs called generalized IFSs. Bhowmik and Pal (2009, 2010) defined generalized interval-valued intuitionistic fuzzy set (GIVIFS) and presented various properties of it.

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J. Li and H. Li (2008) discussed on the cut-sets of fuzzy sets and reveals the relationship between the fuzzy sets and classical sets. Decomposition theorem can be obtained based on the cut-sets. X. Yuan, H. Xing Li, K. Sun (2011) described by neighborhood relation of the fuzzy point and fuzzy set, which has many applications in fuzzy topology and fuzzy algebra.

The organization of this article is as follows. In Section 2, some basic properties of GIVIFS are redefined. In Section 3, different types of interval cut-sets of GIVIFS are defined and some properties of these cut-sets are given. In Section 4, three decomposition theorems on interval cut-sets of GIVIFSs are gained. Finally, in Section 5, a conclusion of this paper is given.

2. PRELIMINARIES

In this section, we recalled some preliminaries and the definition of IVIFS and GIVIFS.

**Definition 2.1:** An IVIFS A over X (universe of discourse) is an object having the form $A = \{ \langle x, M_A(x), N_A(x) \rangle | x \in X \}$, where $M_A(x): X \rightarrow [I]$ and $N_A(x): X \rightarrow [I]$. The intervals $M_A(x)$ and $N_A(x)$ denote the intervals of the degree of membership and degree of non-membership of the element $x$ to the set $A$, where $M_A(x) = [M_A^L(x), M_A^U(x)]$ and $N_A(x) = [N_A^L(x), N_A^U(x)]$, for all $x \in X$, with the condition $0 \leq M_A^U(x) + N_A^U(x) \leq 1$.

For simplicity, we denote $A = \{ \langle x, [A^L(x), A^U(x)], [B^L(x), B^U(x)] \rangle | x \in X \}$.

**Definition 2.2:** If the IVIFSA = $\{ \langle x, MA(x), NA(x) \rangle | x \in X \}$, satisfying the condition $MAU(x) \land NAU(x) \leq 0.5$ for all $x \in X$ then $A$ is called generalized interval-valued intuitionistic fuzzy set (GIVIFS). The condition $M_{AU}(x) \land N_{AU}(x) \leq 0.5$ is called generalized interval-valued intuitionistic fuzzy condition (GIVIFC). The maximum value of $MAU(x)$ and $N_{AU}(x)$ is 1.0, therefore GIVIFC imply that $0 \leq MAU(x) + N_{AU}(x) \leq 1.5$.

It may be noted that all IVIFS are GIVIFS but the converse is not true.

Let $F(X)$ be the set of all GIVIFSs defined on $X$.

2.1. Some Operations on GIVIFSs

In [2], Bhowmik and Pal defined some relational operations on GIVIFSs. Let $A$ and $B$ be two GIVIFSs on $X$, where

$A = \{ \langle [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] \rangle | x \in X \}$

and

$B = \{ \langle [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)] \rangle | x \in X \}$.

Then,

1. $A \subseteq B$ iff $\{ (M_{AU}(x) \leq M_{BU}(x) \text{ and } M_{AL}(x) \leq M_{BL}(x)) \}$

$\{ (N_{AU}(x) \geq N_{BU}(x) \text{ and } N_{AL}(x) \geq N_{BL}(x)) \}$, for all $x \in X$.

2. $A \cap B = \{ \langle \min\{M_{AL}(x), M_{BL}(x)\}, \min\{M_{AU}(x), M_{BU}(x)\}\rangle | x \in X \}$

$[\max\{N_{AL}(x), N_{BL}(x)\}, \max\{N_{AU}(x), N_{BU}(x)\}]$: $x \in X$.

3. $A \cup B = \{ \langle \max\{M_{AL}(x), M_{BL}(x)\}, \max\{M_{AU}(x), M_{BU}(x)\}\rangle | x \in X \}$

$[\min\{N_{AL}(x), N_{BL}(x)\}, \min\{N_{AU}(x), N_{BU}(x)\}]$: $x \in X$.

3. INTERVAL CUT-SETS ON GIVIFS AND SOME RESULTS

In [8], Wang and Jin has introduced some kinds of cut-sets for interval-valued fuzzy sets based on fuzzy interval and interval-valued fuzzy sets. Here we define some types of interval cut-sets for GIVIFS.
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