Chapter 10
Further Investigation of the Period–Three Route to Chaos in the Passive Compass–Gait Biped Model

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ABSTRACT
This chapter presents further investigations into the period-three route to chaos exhibited in the passive dynamic walking of the compass-gait biped robot as it goes down an inclined surface. This discovered kind of route in the passive bipedal locomotion was found to coexist with the conventional period-one passive hybrid limit cycle. The further analysis on the period-three route chaos is realized by means of the Lyapunov exponents and the fractal Lyapunov dimension. Numerical computation method of these two tools is presented. The first return Poincaré map of the chaotic attractor and its basin of attraction are presented. Furthermore, the further study of the period-three passive gait is realized. The analysis of the period-three hybrid limit cycle is given. The balance between the potential energy and the kinetic energy of the biped robot is illustrated. In addition, the basin of attraction of the period-three passive gait is also presented.

1. INTRODUCTION
Robotics has evolved in recent years in various fields such as industrial robotics, medical robotics, domestic robotics, military robotics, etc. One of the most interesting applications in robotics is the analysis

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of human walking through various prototypes of biped robots. The studies have been found to solve some problems related to the stability of human walking and also related to the design of active and passive prostheses of lower limbs of the human being. However, despite its simplicity, human walking is considered quite complex from a dynamic system point-of-view and it is not very well understood. The dynamic walking of biped robots is modeled by an impulsive hybrid nonlinear dynamics. To obtain a synergy between human walking gaits study and biped robots, a simple two-link bipedal mechanism will be a good basic experimental as well as theoretical model.

In the concept of bipedal robotics, passive dynamic walking has attracted the attention of many researchers and has been considered as the starting point for the control of biped robots. Passive dynamic models of biped walking have proven useful in understanding generalized principles that govern walking motions. The term passive dynamics arises from the ability of these models to walk without active control. Thus, passive dynamic walking is a mode of bipedal locomotion for which the biped robot requires no exogenous source of energy but it only uses gravity to walk on an inclined plane (McGeer, 1990; Goswami et al., 1996, 1998; Wisse et al., 2004; Gritli et al., 2012c). This method of walk solves the problem of energy consumption of bipedal robots and get a maximum energetic efficiency. In addition, the use of passive dynamic walking is expected also to obtain additional insights into the design principles of legged locomotion in nature. The best known biped robot using passive dynamic walking is the compass-gait biped robot. This biped is a two-link bipedal mechanism that was originally studied in 1996 by Goswami et al. (1996, 1998). These researchers have shown that this type of bipedal walking can generate chaos and period-doubling bifurcations (Wiggins, 2003) based on certain intrinsic geometric and physical parameters of the biped robot. Until nowadays, many researchers are working on the passive dynamic walking of the compass-gait biped robot and other simple passive biped robots in order to find other properties that can help in the understanding of human walking and also for the control of walking gaits of biped robots. The list of publications is very long. We cite for example (Kaygisiz et al., 2006; Safa et al., 2007; Norris et al., 2008; Kai & Shintani, 2011; Li & Yang, 2012; Li et al., 2013).

One of the most important factors for successful and efficient walking is stability. In fact, the physical biped robots that have been designed around passive dynamic walkers are very sensitive to small perturbations. One of fundamental tools used to investigate stability of dynamic systems is the Lyapunov exponents. Since the fundamental paper of Oseledec (1968), the Lyapunov characteristic numbers or exponents are an important tool for the characterization of dynamical systems attractors of finite-dimensional nonlinear dynamic systems and their initial sensitivity to nearby initial conditions. The spectrum of Lyapunov exponents measures in fact the average divergence or convergence of nearby orbits along certain directions in state space. Sekhavat et al. (2004) employed the concept of Lyapunov exponents in order to analyze the stability of nonlinear dynamical systems and showed that the method is constructive and powerful. The sign of the largest Lyapunov exponent can infer the stability of systems and can rigorously prove the stability of the nonlinear system if numerical artefacts are under control (Kuo, 1995; Sekhavat et al., 2004).

We showed recently that a cyclic-fold bifurcation is generated in the passive dynamic walking of the compass-gait biped giving rise to a cascade of period-doubling bifurcations and then a period-three route to chaos (Gritli et al., 2012c). We have analyzed this period-doubling route to chaos by means of the spectrum of Lyapunov exponents and the fractal dimension (Gritli et al., 2012a). In this chapter, we will revisit the passive dynamic walking of the compass-gait biped robot. We will give further results on the period-three passive limit cycle and the corresponding route to chaos. Our analysis will be based first on the computation of the Lyapunov exponents and the fractal dimension in order to quantify order