Chapter 11
Bifurcation, Quasi-Periodicity, Chaos, and Co-Existence of Different Behaviors in the Controlled H-Bridge Inverter

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ABSTRACT
This chapter deals with the analysis of the dynamic behavior of a controlled single-phase H-bridge inverter. The authors show that in addition to border collision bifurcation, when it is controlled with a time-delayed controller or with a dynamic controller that increases the system dimension, the H-bridge inverter can exhibit several other types of behaviors such as Neimark-Sacker bifurcation, quasi-periodicity, and coexistence of different periodic behaviors, as well as coexistence between periodic and chaotic behaviors. The proposed controllers are of different types. In addition to the Fixed-Point Induced Controller (FPIC), the authors also present the Time-Delayed Feedback Controller (TDFC) and the dynamic linear controller, such as the proportional-integral controller. The main issue of this chapter is to perform analysis within and beyond the stability region. Analytic calculation and numerical simulations are presented to confirm the obtained results.

1. INTRODUCTION
Power electronics is a discipline that has emerged from the need to convert electrical energy. Its field of application is wide and concerns industrial, commercial, residential and also aerospace environments. Nowadays, there is a great interest to use green energy such as the solar and wind energy, there is also a need to use mobile electrical devices supplied by batteries or fuel cells. Knowing that such power sources supply a low DC voltage, then the design of electronic devices that convert the low DC signal...
Bifurcation, Quasi-Periodicity, Chaos, and Co-Existence of Different Behaviors

into a high AC signal similar to power that would be available at an electrical wall outlet becomes a challenging endeavor. This prescribed aim has been achieved using two stages. The first one is a boosting DC to DC converter and the second is the DC to AC inverter (Luo and Ye, 2004; Rashid, 2001). Today, there are a myriad of such power converters ranging from the most basic to the most complicated and yielding to several optimal behaviors in different senses such as wave forms, power efficiency and circuit simplicity and size.

Power converters are the main devices that helped boosting the use of electronic devices. However, converters have been used long before their behavior is completely understood. Indeed, it’s only on late eighties that Hamill (Hamill and Jefferies, 1988) reported the existence of several nonlinear phenomena in power converters. These include subharmonic oscillations, quasi-periodic operations, bifurcations and chaos.

Chaos in power electronics have intrigued many researchers (Di Bernardo and Tse, 2002; Deane and Hamill, 1990; Tse and Di Bernardo, 2002; Robert and Robert, 2002). In fact, engineers have frequently encountered chaos in power electronics systems, but more often than not this phenomenon was considered as strange and undesirable, hence engineers usually attempted to avoid chaos. During the last two decades, tools of analyzing bifurcations and chaos have been well developed. Therefore, the investigation of the very peculiar aspect of this phenomenon has become an attractive endeavor.

Power converters are basic switching circuits that are modeled by a number of linear differential equations corresponding to different topologies. The switching procedure imposed by different control schemes causes toggling among a set of linear circuits. Therefore the overall dynamics can be easily described by a piecewise linear model. In the case of simple linear models, a discrete map can be obtained by solving and stacking up solutions (Di Bernardo and Vasca, 2000; Robert and El Aroudi, 2006). The discrete-time modeling of the converters has provided an easy way to conceive controllers by using the discrete-time control theory and that can be applied further using the digital pulse width modulator (DPWM) (Feki et al., 2004; Robert et al., 2003; Hamza et al., 2011; Kaoubaâ et al., 2012).

Controlling the converters has many objectives such as ensuring converter closed loop stability and enhancing efficiency and dynamic performances. The switching nature of the continuous time mathematical model or the discrete time model with saturating controller (duty cycle) makes the control of the electric power converters a challenging topic in itself. In addition, the converter performances are tightly related to the adopted control method. Therefore, the control of electric power converters have attracted the attention of many researchers (Pinard, 2007; Rodriguez et al., 2000; Alvarez-Ramirez and Espinosa-Pérez, 2002).

In addition to linear control methods (Robert and Robert, 2002; Di Bernardo and Tse, 2002), several nonlinear techniques have been used to control the DC/DC and DC/AC converters as well. For instance, passivity based controllers (Sira-Ramirez et al., 1997), sliding mode controllers (Bensaada and Stambouli, 2013; Fernandez-Vargas and Ledwich, 2010; Tan et al., 2005, 2007; Wai and Shih, 2011), adaptive controllers (Albea et al., 2011), robust controllers (Olalla et al., 2011) and optimal controllers (Kawashima et al., 2012). The non conventional controllers such as neural controllers (Chen et al., 2012; Gnanasaravanan and Rajaram, 2012a), genetic algorithm (Gnanasaravanan and Rajaram, 2012b; Miladi et al., 2014) and fuzzy controllers (Baek and Park, 2012; Mazouz and Midoun, 2011; Nachidi et al., 2013; Guesmi et al., 2008) have also been used to control the power converters.

In this chapter we present an analysis of the dynamic behavior of a current mode controlled single phase H-bridge inverter driving a resistive and inductive load under the action of simple controller methods. We show that the switching nature of the converter provides it with a variety of complex be-