Chapter 19

Discrete–Time Approximation of Multivariable Continuous–Time Delay Systems

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ABSTRACT

Many works are related to the analysis and control of either continuous or else discrete time-delay systems. In general, discrete-time controls have become more and more preferable in engineering because of their easy implementation and simple computation. However, the available discretization approaches for the systems having time delays increase the system dimensions and have a high computational cost. The case studies in this chapter support the efficiency of the two methods. However, the discretization of continuous time-delay systems has not been sufficiently/extensively studied in many works. In this work, the authors present two methods of the effective discretization approach for the continuous-time systems with an input and output delays. Sampled-data time-delay systems with internal and external point delays are described by approximate discrete time-delay systems in the discrete domain. These approximate discrete systems allow the hybrid control of time-delay systems.

1. INTRODUCTION

Systems with delays abound in the world. One reason is that nature is full of transparent delays. Another reason is that time-delay systems are often used to model a large class of systems with delays that frequently appearing in engineering. Typical examples of time-delay systems are communication networks, chemical processes, teleoperation systems, biosystems, and underwater vehicles and so on. The presence

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Discrete-Time Approximation of Multivariable Continuous-Time Delay Systems

of delays makes system analysis and control design much more complicated. The emphasis is on systems with a single input/output delay although the delay-free part of the plant can be multi-input, multi-output (MIMO) when the delays in different channels are the same. The major tools used in this chapter are State-transition method and a Method based on the trapezoidal rule for integration.

With the pace of development in the field of microelectronics, analog controllers yield more places for their digital computers. Indeed, and given the importance of these control systems, it uses methods and numerical models to analyze and/or control industrial processes. To implement such a control structure and ensure the desired objectives, modeling in discrete-time analog systems is required.

Two types of representation are available to model continuous or discrete systems namely the external representation that uses input-output (or transfer function matrix) relations and the internal representation of dynamic system which is based on the concept of state.

Digital control of physical systems usually requires developing discrete models. Several modeling strategies, developed in the literature reflecting a meaningful description of dynamical systems to study, led to mathematical tools generally resulting in linear or nonlinear systems with or without delays whose behavior is close to the real system models. These models, which are described by relations between input and output variables, can be modified by inputs considered secondary (disturbances) that still exist in practice. Knowing the benefits given by the state description of dynamical systems, opt for following such descriptions.

A discrete time delay system is defined as an operator between two discrete-time signals that involve a time called delays. A physical system for which \( u(k) \) is the general term of the sequence input and \( x(k + 1) \) is the general term of the sequence and state of delayed \( h \).

Initial modeling of a system to discrete time delays often leads to writing a recurrent equation between different terms of the input and output sequences. This formulation of the recursive equation is well suited for numerical calculation. This is the form in which these algorithms are control methods. The system is fully defined and recurrent equation can be solved if the initial conditions are specified. These forms can be simplified mathematically using the lag operator and allow formalizing recurrent equations as follows:

\[
x(k + 1) = Ax(k) + Bx(k - q) + Cu(k); h = qT
\]

where \( A, B \) and \( C \) are matrices of suitable dimensions, and \( q \) is assumed to be constant delay.

Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect (Chen & Latchman, 1994; Shinskey, 1967). Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission. The value of the delay is determined by the distance and the transmission speed. Some delays are short, others are very long. The presence of long delays makes system analysis and control design much more complex. What is worse is that some delays are too long to perceive and the system is misperceived as one without delays.

The presence of time-delays in a practical system may induce instability, oscillation and poor performance and complicates the analysis and design of feedback controllers. Control systems with time-delays exhibit complex behavior because of their infinite dimensionality, even in the case of linear time-invariant systems that have constant time-delays in the input or states have infinite dimensionality when expressed...