Chapter 8
Formalization of Hidden Markov Model

ABSTRACT

In this chapter, the authors provide the formalization of extended DTMC models, namely Hidden Markov Models (HMMs), which are the core concept for formally evaluating the probability of the occurrence of a particular observed sequence and finding the best state sequence to generate given observation (Mantyla & Tutkimuskescus, 2001; Rabiner, 1990). In order to present the usefulness of the formalization of HMM and the formal verification of HMM properties, the authors illustrate the formal analysis of a DNA (Deoxyribon Nucleic Acid) sequence at the end of the chapter.
8.1 DEFINITION OF HMM

In order to accurately analyze the HMMs (Eddy, 2004), we propose to apply the formalized DTMC to formally define HMMs and verify their properties in higher-order logic as the extended DTMC models.

An HMM is a pair of two stochastic processes \{ X_k; Y_k \} k ≥ 0, where \{ X_k \} k ≥ 0 is a Markov chain, and \{ Y_k \} k ≥ 0 is conditionally independent of \{ X_k \}, i.e., Y_k depends only on X_k and not on any X_t, such that t ≠ k. The HMMs model situations where an experimenter sees some observers at every instant (mathematically represented by Y_k) and suspects these observables to be the outcome of a process that can be modeled by a Markov chain (\{ X_k \} k ≥ 0). The name “Hidden Markov Model” arises from the fact that the state in which this model is at a particular instant is not available to the observer. Now, a HMM is defined as a parameterized triple (A, B, π (0)) with the following conditions:

1. Hidden Markov Chain \{ X_k \} k ≥ 0 with a finite state space S, the initial distribution \( \pi (0) = \{ \pi_i (0) \} i ∈ S \) and the transition probabilities \( A = \{ a_{ij} \} i ∈ S, j ∈ S \).
2. A random process \{ Y_k \} k ≥ 0 with finite state space O. The hidden Markov chain and the random process are associated with the emission probabilities \( B = \{ b_j (O_k) \} j ∈ S, k ∈ O = \{ \Pr \{ Y_n = O_k \mid X_n = j \} \} j ∈ S, k ∈ O \). It implies that:
   a. \( \forall j k. b_j (O_k) ≥ 0 \),
   b. \( \sum k ∈ Obj (O_k) = 1 \).
3. The random process \{ Y_k \} k ≥ 0 and hidden Markov chain \{ X_k \} k ≥ 0 have conditional independence.

This yields the formalization (Liu, 2013):

**Definition 8.1**

\[ ⊢ ∀ X Y p s_x s_y p_0 p_{ij} p_{xy} . \]

\[ \text{hmm } X Y p s_x s_y p_0 p_{ij} p_{xy} = \]

\[ \text{dtmc } X p s_x p_0 p_{ij} ∧ (∀ t. \text{random variable } (Y t) p s_y) ∧ \]

\[ (∀ i. i ∈ \text{space } s_y ⇒ \{ i \} ∈ \text{subsets } s_y) ∧ \]

\[ (∀ t a i. P \{ x \mid X t x = I \} ≠ 0 ⇒ \]

\[ P \{ (x \mid Y t x = a) \mid \{ x \mid X t x = I \} \} = p_{xy} t a I \} ∧ \]

\[ ∀ t a i t x0 ty0 sts_x sts_y ts_x ts_y . \]

\[ t ∉ \{ tx0 + m \mid m ∈ ts_x \} ∧ t ∉ \{ ty0 + m \mid m ∈ ts_y \} ∧ \]

\[ P \{ (x \mid X t x = I \} ∧ k ∈ t s_x \{ x \mid X (tx0 + k) x = EL k sts_x \} \}

\[ k ∈ t s_y \{ x \mid Y (ty0 + k) x = EL k sts_y \} \} ≠ 0 ⇒ \]
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