Chapter 2
Coalition Formation Game for Wireless Communications

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ABSTRACT
With the emergence of cooperation as a new communication paradigm, and the need for self-organizing, distributed networks, it is necessary to seek suitable game theoretical tools that allow to analyze and study the behavior and interactions of the nodes in future communication networks. In this context, this chapter introduces the coalition formation game theory, and its potential applications in communication and wireless networks. Specifically, it presents the fundamental components, the key properties, the mathematical techniques, the solution concepts, and describes the methodologies for applying these games in several applications drawn from the state-of-the-art research in communications.

INTRODUCTION
As we know, there has been significant growth in research activities, which exploit game theory to model and analyze the communication networks, and help the network devices to make independent and rational strategic decisions by low complexity distributed algorithms. In general, game theory can be divided into two branches, namely, non-cooperative and cooperative game theory. The former provides effective tools for studying the competitive behavior of rational players, and its potential applications in communication and wireless networks become more perfect. The latter focuses on collaborative scenarios. In particular, the coalition formation games are the main branch of cooperative games, which describes the formation of cooperating groups of players, referred to as coalitions, for the purpose of strengthening the players’ positions in a game.

Note that, it is a trend that cooperation emerges as a new networking paradigm, which is capable of improving the performance from the physical layer up to the network layer. However, implementing cooperation in large scale, self-organizing, and distributed networks needs to face several obstacles, e.g., ...
modeling, flexibility, scalability, optimality, and complexity. In this context, this chapter focuses on the coalition formation game theory to deal with such obstacles. Unfortunately, there are only several works which are restricted to study very limited aspects of cooperation in networks.

This chapter aims to overview the state-of-the-art research advances, from coalition formation games to their communication applications. Firstly, the fundamental components are presented, including the key properties, the mathematical techniques of coalition formation games. Then, two applications that are drawn from the state-of-the-art research in communications are introduced to describe the methodologies for applying these games. In particular, they address the major challenges in applying coalition formation games to the understanding and designing of communication systems, with emphasis on both new analytical techniques and novel application scenarios (Saad, Han, Debbah, Hjørungnes, & Baar, 2009a). Finally, the future application-oriented approaches to coalition formation games are discussed, and the available research directions for using this strong analytical tool of coalitional games are further pointed out.

INTRODUCTION TO COALITION FORMATION GAMES

In essence, a coalition formation game is defined as a triplet \((\mathcal{N}, v, \mathcal{S})\). Specifically, \(\mathcal{N} = \{1, \ldots, N\}\) denotes a set of players, which try to form coalitions to strengthen their positions in the game. Any coalition represents an agreement among the players in the same coalition to act as a single entity. \(v\) denotes the coalition value, which quantifies the worth of a coalition in a game. The definition of the coalition value determines the form and type of the game. In particular, \(\mathcal{S}\) is a coalitional structure or a partition of \(\mathcal{N}\), i.e., a collection of coalitions \(\mathcal{S} = \{S_1, \ldots, S_i\}\), such that \(\forall \neq k, S_i \cap S_k = \emptyset\), and \(\bigcup_{i=1}^{\infty} S_i = \mathcal{N}\). It is imposed by an external factor on the game, such as physical restrictions in the problem.

The most common form of a coalition formation game is the characteristic form, where the value of a coalition just depends on the members of this coalition. As such, the coalition value \(v\) can be treated as a characteristic function, and further, the characteristic function can be either transferable utility (TU) or nontransferable utility (NTU). More precisely, the value of a game in characteristic form with TU is a function over the real line defined as \(v : 2^{\mathcal{N}} \rightarrow \mathbb{R}\). It associates with each coalition \(S_i \subseteq \mathcal{N}\) a real number quantifying the values of \(S_i\). In other words, the coalition values in TU games are treated as monetary values which are distributed among the members in a coalition by using an appropriate fairness rule. The amount of utility that a player \(j \in S_i\) obtains from the division of \(v(S_i)\) constitutes the utility of player and is denoted by \(x_j\). The vector \(x \in \mathbb{R}^{\|\mathcal{S}\|}\), whose element \(x_j\) is the utility of player \(j \in S_i\), shows a utility allocation. Here, \(|\cdot|\) represents the cardinality of a set. In addition, the coalition values in NTU games can not be assigned a single real number, or rigid restrictions exist on the distribution of the utility. Taking coalition \(S_i\) in an NTU game for example, its value, \(v(S_i)\) is no longer a function over the real line, but a set of utility vectors, \(v(S_i) \subseteq \mathbb{R}^{\|\mathcal{S}\|}\), where each element \(x_j\) of a vector \(x \in v(S_i)\) represents a utility that player \(j \in S_i\) can obtain within coalition \(S_i\) given a certain strategy selected by \(j\) while being a member of \(S_i\). According to this definition, a TU game can be regarded as a particular case of the NTU framework. Coalition formation games in characteristic form with TU or NTU are one of the most important types of games.