Representing an Intrinsically Nonmetric Space of Compass Directions in an Artificial Neural Network

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ABSTRACT

The purpose of the article is to train an artificial neural network to make a judgment that is intrinsically antisymmetric, and to determine how such a judgment is mediated by the network’s internal representations. One key component of navigation is judging the distance from one location to another. A second key component of navigation is judging heading — that is, judging the direction from one location to another. Importantly, heading judgments do not preserve the metric properties of space. In particular, they are antisymmetric: the judged heading from location x to location y should be opposite to the judged heading from location y to location x. What kind of representation can mediate such nonmetric navigational judgments? To explore this question, we trained an artificial neural network to judge the various bearings amongst 13 different cities in Alberta. We then interpreted the internal structure of this network in order to determine the nature of its internal representations. We found that the artificial neural network had developed a coarse directional code, and that one of the advantages of such coding was its ability to represent antisymmetric spatial regularities.

Keywords: neural networks; spatial representation

INTRODUCTION

Cognitive informatics is an interdisciplinary field of research in which formal aspects of information processing problems are related to biological mechanisms involved in such processing (Wang, 2003). One kind of information processing medium of interest to cognitive informatics is that of artificial neural networks (e.g., Anderson, 2003; Dawson & Zimmerman, 2003). The current article illustrates an example of artificial neural network research in the cognitive informatics tradition. The formal property of interest is the representation of relations that are intrinsically antisymmetric. At issue are the kinds of internal representations developed by neural networks to deal with this formal property.

Representations that preserve the metric properties of space have been fundamentally important to the study of how humans and ani-
mals navigate (Kitchin, 1994). Behavioral studies have demonstrated that animal representations of space do indeed appear to preserve a good deal of its metric nature (for introductions, see Cheng & Spetch, 1998; Gallistel, 1990; Gallistel & Cramer, 1996). Since the discovery of place cells in the hippocampus (O’Keefe & Dostrovsky, 1971; O’Keefe & Nadel, 1978), many researchers have been concerned with identifying the biological substrates that encode metric space (Redish, 1999).

If a space is metric, then relationships between points in the space are constrained by three different principles (Blumenthal, 1953). The first is the minimality principle, which dictates that the shortest distance in the space is between a point \( x \) and itself. The second is the symmetry principle, which dictates that the distance in the space between two points \( x \) and \( y \) is equal to the distance between points \( y \) and \( x \). The third is the triangle inequality, which dictates that the shortest distance in the space between two points \( y \) and \( x \) is a straight line. The fourth is the non-negativity principle, which dictates that every distance in the space between two points \( y \) and \( x \) must be greater than or equal to 0.

The assumption that some mental representations are metric spaces has had important theoretical and methodological impacts on cognitive science. With respect to theory, metric spaces have been used to explain a variety of cognitive phenomena, including models of similarity judgments (Shepard, 1974), analogical reasoning (Sternberg, 1977), judgments of metaphor aptness (Tourangeau & Sternberg, 1981, 1982), and transformations of mental images (Shepard, 1984). With respect to method, the assumption that some cognitive judgments are constrained by metric properties has led researchers to analyze a variety of behaviours using multidimensional scaling (Romney, Shepard & Nerlove, 1972; Shepard, Romney & Nerlove, 1972).

### Navigation and Nonmetric Relationships

Importantly, the principles that characterize a metric space are all defined as properties of distance. Knowing the distance from a current location to a goal location is obviously an important component of navigation. However, knowing this distance is not by itself sufficient for successful navigation. In addition, it is also crucial to know the direction in which to travel: the bearing of the goal location from the current location. Interestingly, the notion of direction does not fit well with the metric properties of space. Consider a place on the map of Alberta, the city of Calgary. The town of Banff is west of Calgary. But this directional relationship violates the symmetry property of a metric space, because the direction from Banff to Calgary is east, not west. Indeed, For example, if one were to create a table of directions between cities, representing these directions as cosines, this table would be an antisymmetric matrix. That is, the value recorded in row \( x \) and column \( y \) of the table would be equal to the negative of the value recorded in row \( y \) and column \( x \) of the same table for every pair of off-diagonal cells. Given their intrinsically antisymmetric nature, how might a navigating system represent the bearings between pairs of locations? Of course, one possibility would be to represent the positions of the locations in a metric space, and then compute the required bearings.

However, it is not clear that this approach is appropriate for biological systems. O’Keefe and Nadel (1978) argued that the hippocampus represents the metric properties of space in a cognitive map. Importantly, a number of researchers have argued against this position, and against the proposal that the hippocampus provides a metric, systematic, and cohesive cognitive map. First, place cells are not organized topographically; the arrangement of place cells in the hippocampus is not isomorphic to the arrangements of locations in an external space (Burgess, Recce & O’Keefe, 1995; Eichenbaum, Dudchenko, Wood, Shapiro & Tanila, 1999; McNaughton, Barnes, Gerrard, Gothard, Jung & Knierim, 1996).