INTRODUCTION

Decision making is a process that chooses a preferred option or a course of actions from among a set of alternatives based on certain criteria. The study on decision making is interested in multiple disciplines such as cognitive informatics, computer science, management science, economics, sociology, psychology, political science, and statistics. A number of decision strategies have been proposed from different angles and application domains such as the maximum expected utility and Bayesian method. However, there is still a lack of a fundamental and mathematical decision model and a rigorous cognitive process for decision making. This article presents a fundamental cognitive decision making process and its mathematical model, which is described as a sequence of Cartesian-product based selections. A rigorous description of the decision process in real-time process algebra (RTPA) is provided. Real-world decisions are perceived as a repetitive application of the fundamental cognitive process. The result shows that all categories of decision strategies fit in the formally described decision process. The cognitive process of decision making may be applied in a wide range of decision-based systems such as cognitive informatics, software agent systems, expert systems, and decision support systems.

Keywords: cognitive informatics; cognitive process; decision-making; expert systems; formal description; mathematical model; RTPA; software engineering
making process in the brain, which may be served as the foundation of various decision making theories.

Decision theories can be categorized into two paradigms: the descriptive and normative theories. The former is based on empirical observation and on experimental studies of choice behaviors; and the latter assumes a rational decision-maker who follows well-defined preferences that obey certain axioms of rational behaviors. Typical normative theories are the expected utility paradigm (Osborne & Rubinstein, 1994) and the Bayesian theory (Berger, 1990; Wald, 1950). Edwards developed a 19-step decision-making process (Edwards et al., 2001) by integrating Bayesian and multi-attribute utility theories. Zachary, Wherry, Glenn, and Hopson (1982) perceived that there are three constituents in decision making known as the decision situation, the decision maker, and the decision process. Although the cognitive capacities of decision makers may be greatly varying, the core cognitive processes of the human brain share similar and recursive characteristics and mechanisms (Wang, 2003a; Wang & Gafurov, 2003; Wang & Wang, 2004; Wang et al., 2004).

This article adopts the philosophy of the axiom of choice (Lipschutz, 1967). The three essences for decision making recognized in this article are the decision goals, a set of alternative choices, and a set of selection criteria or strategies. According to this theory, decision makers are the engine or executive of a decision making process. If the three essences of decision making are defined, a decision making process may be rigorously carried out by either a human decision maker or by an intelligent system. This is a cognitive foundation for implementing expert systems and decision supporting systems (Ruhe, 2003; Ruhe & An, 2004; Wang et al., 2004; Wang, 2007a).

In this article, the cognitive foundations of decision theories and their mathematical models are explored. A rigorous description of decisions and decision making is presented. The cognitive process of decision making is explained, which is formally described by using real-time process algebra (RTPA). The complexity of decision making in real-world problems such as software release planning is studied, and the need for powerful decision support systems are discussed.

A MATHEMATICAL MODEL OF DECISIONS AND DECISION MAKING

Decision making is one of the fundamental cognitive processes of human beings (Wang et al., 2004; Wang, 2007a; Wang, 2007b) that is widely used in determining rational, heuristic, and intuitive selections in complex scientific, engineering, economical, and management situations, as well as in almost each procedure of daily life. Since decision making is a basic mental process, it occurs every few seconds in the thinking courses of human mind consciously or subconsciously.

This section explores the nature of selection, decision, and decision making, and their mathematical models. A rigorous description of decision making and its strategies is developed.

The Mathematical Model of Decision Making

The axiom of choice (or choice) (Lipschutz, 1967) states that there exists a selection function for any nonempty collection of nonempty disjoint sets of alternatives.

Definition 1. Let \( \{A_i \mid i \in I\} \) be a collection of disjoint sets, \( A_i \subseteq U \), and \( A_i \neq \emptyset \), a function

\[
c: \{A_i\} \rightarrow A_i, \quad i \in I
\]  

(1)

is a choice function if \( c(A_i) = a_i, \quad a_i \in A_i \). Or an element \( a_i \in A_i \) may be chosen by \( c \), where \( A_i \) is called the set of alternatives, \( U \) the universal set, and \( I \) a set of natural numbers.

On the basis of the choice function and the axiom of selection, a decision can be rigorously defined as follows.