A Virtual Laboratory of Mathematics Education

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**INTRODUCTION**

Virtual laboratories are increasing for all areas of scientific domains. However, the concept of such laboratories can be extended to include educational activities. But the point is, how to do it?

We have been involved in the development of a virtual laboratory on mathematics education for the past 15 months. Our institution is involved in the training of primary school teachers. On their final year, our students go to primary schools and teach there in some supervised classes. Our idea is to create an instrument that can be used with primary school children by our students, during the earlier-mentioned practice in schools. Therefore it has to be an instrument with features that include being appealing to children, easy to use, and helpful to novice primary school teachers.

**WHAT IS OUT THERE?**

Virtual laboratories are very popular especially within the field of experimental sciences such as chemistry, physics, biology, or even medicine. On a quick search through the Internet, one can find lots of implementations, in different areas, of so-called virtual laboratories. These laboratories allow the performance of experiments that are difficult or even impossible to implement in real laboratories, either for economical reasons or for ethical reasons (such as experiments involving live animals).

Also, virtual laboratories offer the possibility of verification/simulation of physical laws and allow variations within the same experiment and the realization of experiments in different levels of complexity. In this sense we can even say that virtual laboratories become more powerful than the real ones.

**WHAT IS IN THERE?**

As far as mathematics are concerned, one can also find several virtual laboratories of different kinds. Even though mathematics is not an empirical science, it uses observation, experimentation, induction, comparisons and generalizations. However, the objects of mathematical investigation are purely mental and not real. In mathematics, concepts are abstract and are totally controlled by their definition, unlike empirical sciences where concepts tend to approximate a given reality (Fischbein, 1996). In this sense we can speak about mathematical virtual laboratories. Let us note that we can find so-called virtual labs that are nothing more than presentations of mathematical content in a virtual media and therefore cannot be considered, in our perspective, laboratories. We assume that in order to be a laboratory it should include some kind of experiment and/or interaction within the virtual environment.

Searching the Internet, we were able to find lots of virtual laboratories which we organized in the following four types:

- Labs devoted to the learning of “advanced” topics of mathematics with specific software designed to perform function analysis, make graphics, solve equations, and so forth; these labs are intended usually for university students or for mathematicians who can gather information/data for the understanding of a topic, the proof of a theorem, or the formulation of a theory. They also offer the possibility of exploring mathematical concepts and modelling.

- Labs that allow the communication of users emerged in an e-learning environment. These labs use the Internet as a means of communication between different people in several places. Usu-
ally these labs present a series of tasks to solve and users can interchange solutions, ideas, discuss extensions, and so forth. For example, there is a European project called WebLabs (http://www.weblabs.eu.com/) that investigated creating new ways of representing and expressing mathematical and scientific knowledge in European communities of young learners (10-14 year-olds). The focus of this project was on collaborative construction, description, and interpretation of how things work.

- Labs dedicated to mathematics education, with mathematical materials already prepared for individual use or for class use, with the contents previously separated by schooling years. These labs function as a resource centre where both students and teachers can get suggestions for activities, materials (hands-on), help (both scientific and pedagogic), and so forth.

- Finally, there are labs containing virtual manipulatives which allow the user to virtually manipulate the materials and carry out experiments. For example, we can find the National Library of Virtual Manipulatives, where there are several manipulatives and activities proposed, already separated both by topic/content, by school year, and by manipulative (site: http://nlvm.usu.edu/en/nav/vlibrary.html).

VISIONS ON MATHEMATICS EDUCATION

We can consider two opposing views concerning mathematics education. We shall expose them and build our own position from this opposition.

Acquisition of Knowledge

The first divider for the visions on mathematics teaching has to do with how we consider acquisition of knowledge.

Thorndyke’s connectionism theory, which later was reinforced by Skinner’s behaviourism, followed the principle of the tabulae rasaë, which implies transmission of knowledge from the teacher (Orton, 1992).

Constructivism defends a different thing: every learner constructs his/her knowledge. However, it is widely assumed that even in mathematics, there is knowledge of a social origin and knowledge of a logical type (Nunes & Bryant, 1996).

It is possible to consider extreme versions of constructivism that take this division as artificial. For them all knowledge is socially originated, the conclusion being that all knowledge can be constructed in the classroom (Ernest, 1991).

Hewitt (1999) defends that we should divide the curriculum between those things that are arbitrary (and therefore students must be informed) and those necessary (and students must construct them, otherwise they will memorize without understanding).

This is also our opinion: that mathematics curriculum should be divided between those things that have to be transmitted by the teacher (number names, algorithms, etc.) and those that have to be constructed by the students.

Mathematical Tasks: Exercises or Problems?

In the beginning of the 20th century, Thorndyke stipulated that exercises were the central part of mathematics teaching. For a considerable number of years, exercises were so valued that even nowadays it is hard to find classes where they are not present (Kilpatrick, 1992).

It is with Pólya (1957) that problem solving arises. Today it is included in most curricula around the world.

However, as Schoenfeld (1992) warns, some caution is necessary because many of the announced problem solving is actually exercise solving. Many people have just changed the name but not the nature of the tasks they propose.

In a so-called traditional view, exercise solving is the fundamental component. However, we defend that even though exercises have a place in mathematics teaching, it is problem solving that should be the fundamental component.

Products/Processes

The traditional view on mathematics education stresses the correction of products, achieved rapidly and with a unique procedure, the one considered to be easiest and the fastest.