Chapter II

State Transition Dynamics: Basic Concepts and Molecular Computing Perspectives

Vincenzo Manca, University of Verona, Italy
Giuditta Franco, University of Verona, Italy
Giuseppe Scollo, University of Verona, Italy

ABSTRACT

Classical dynamics concepts are analysed in the basic mathematical setting of state transition systems where time and space are both completely discrete and no structure is assumed on the state’s space. Interesting relationships between attractors and recurrence are identified and some features of chaos are expressed in simple, set theoretic terms. String dynamics is proposed as a unifying concept for dynamical systems arising from discrete models of computation, together with illustrative examples. The relevance of state transition systems and string dynamics is discussed from the perspective of molecular computing.
INTRODUCTION

A dynamical system is a structure that changes in time. This characterization immediately shows the wide range of such a notion. A ball that is moving, a flow of electrons in a conductor, the propagation of a wave, a chemical reaction, and even a chess game, a process running on a computer, or the development and the life of an organism are examples of dynamical systems. What is essential in these systems are three components: space, collecting all the possible states of the system; time, collecting the different instants at which the system is considered; and dynamics, which associates, to each instant, the corresponding state of the system. The notion of state is to be viewed in a wide sense — it could be a set of microstates, or a probability distribution on a set of states, or, as in the case of a quantum systems, a superposition of states expressed by a vector in a suitable vector space. In all cases, the general structure of such a system is given by a triple \( D = (T, S, \delta) \), where the dynamics \( \delta \) is a function from the set \( T \) of instants to the set \( S \) of states. The various kinds of dynamical systems are essentially determined by the structure of the space, the nature of the time, and the way dynamics is characterized.

In the classical theory of dynamical systems suggested by physics, and naturally described by differential equations, time is a continuous entity and instants are the points of the real axis. When the instants are identified with natural or integer numbers, then the systems are usually called discrete dynamical systems, in contrast with the former ones, which are called continuous dynamical systems. But, in both cases, space is always assumed to be a continuous metric space (the usual three-dimensional space, or a Hilbert phase space of possibly infinite dimension). In this paper, we address the problem of considering, in general terms, dynamical systems that are completely discrete in that, not only are the instants natural or integer numbers, but also the space is a discrete entity. We know that any discrete information can be encoded by a string over a suitable alphabet \( V \); therefore, generally, a discrete space is identified by a subset of \( V^* \) (the set of all the strings over an alphabet \( V \)).

We are typically interested in the behaviour of dynamical systems — that is, how their configuration changes in time. When the dynamics is given explicitly, then the behaviour of the system is completely described by it, and, in more precise terms, a triple time-space-dynamics determines a global dynamical system. But, in almost all cases, the dynamics is given implicitly because we only have some conditions \( \Phi(\delta) \) that specify it. Therefore, the main problem to solve is just the computation of the values of the dynamics in time, starting from an initial instant and using local conditions that implicitly determine
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