Chapter III
Generalized Differential Evolution for Constrained Multi–Objective Optimization

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ABSTRACT

Multi-objective optimization with Evolutionary Algorithms has been gaining popularity recently because its applicability in practical problems. Many practical problems contain also constraints, which must be taken care of during optimization process. This chapter is about Generalized Differential Evolution, which is a general-purpose optimizer. It is based on a relatively recent Evolutionary Algorithm, Differential Evolution, which has been gaining popularity because of its simplicity and good observed performance. Generalized Differential Evolution extends Differential Evolution for problems with several objectives and constraints. The chapter concentrates on describing different development phases and performance of Generalized Differential Evolution but it also contains a brief review of other multi-objective DE approaches. Ability to solve multi-objective problems is mainly discussed, but constraint handling and the effect of control parameters are also covered. It is found that GDE versions, in particular the latest version, are effective and efficient for solving constrained multi-objective problems.

INTRODUCTION

During the last two decades, Evolutionary Algorithms (EAs) have gained popularity since EAs are capable of dealing with difficult objective functions, which are, for example, discontinuous, non-convex, multimodal, and nondifferentiable. Multi-objective EAs (MOEAs) have also gained
popularity since they are capable of providing multiple solution candidates in a single run that is desirable with multi-objective optimization problems (MOPs).

Differential Evolution (DE) is a relatively new EA and it has been gaining popularity during previous years. Several extensions of DE for multi-objective optimization have already been proposed. The simplest approaches just convert MOPs to single-objective forms and use DE to solve these (Babu & Jehan, 2003; Chang & Xu, 2000; Wang & Sheu, 2000), whereas more recent ones use the concept of Pareto-dominance. The chapter contains a brief review of multi-objective DE approaches as well as constraint handling techniques used with DE.

This chapter concentrates on describing a DE extension called Generalized Differential Evolution (GDE), its development phases, and performance. GDE is a general-purpose optimizer for problems with constraints and objectives. Since different GDE versions differ in their ability to handle multiple objectives, the chapter mainly concentrates on this aspect but it also deals with constraint handling and the effect of control parameters. It is found that GDE versions, in particular the latest version, are effective and efficient for solving constrained multi-objective problems.

The rest of the chapter is organized as follows: In Section BACKGROUND, the concept of multi-objective optimization with constraints is handled briefly. Also, basic DE and its extensions for multi-objective and constrained optimization have been described. Section GENERALIZED DIFFERENTIAL EVOLUTION describes different development phases of GDE with experimental illustrations. Subjects of future work are given in Section FUTURE RESEARCH DIRECTIONS, and finally conclusions are drawn in Section CONCLUSION.

BACKGROUND

Multi-Objective Optimization with Constraints

Many practical problems have multiple objectives and several aspects cause multiple constraints to problems. For example, mechanical design problems have several objectives such as obtaining performance and manufacturing costs, and available resources may cause limitations. Constraints can be divided into boundary constraints and constraint functions. Boundary constraints are used when the value of a decision variable is limited to some range, and constraint functions represent more complicated constraints, which are expressed as functions. A term multi-objective is used when the number of objectives is more than one. A term many-objective is used when the number of objectives is more than two or three (the term is not settled yet).

A constrained multi-objective optimization problem (MOP) can be presented in the form (Miettinen, 1998, p. 37):

\[
\text{minimize } \{f_1(\bar{x}), f_2(\bar{x}), \ldots, f_M(\bar{x})\} \\
\text{subject to } (g_1(\bar{x}), g_2(\bar{x}), \ldots, g_K(\bar{x}))^T \leq 0^T
\]

Thus, there are \( M \) functions to be optimized and \( K \) constraint functions. Maximization problems can be easily transformed to minimization problems and constraint functions can be converted to form \( g_j(\bar{x}) \leq 0 \), thereby the formulation above is without loss of generality.

Typically, MOPs are often converted to single-objective optimization problems by predefining weighting factors for different objectives, expressing the relative importance of each objective. Optimizing several objectives simultaneously without articulating the relative importance of each objective \textit{a priori} is often called Pareto-optimization (Pareto, 1896). An obtained solution