Chapter 34

Efficient Update Control of Bloom Filter Replicas in Large Scale Distributed Systems

Yifeng Zhu
University of Maine, USA

Hong Jiang
University of Nebraska – Lincoln, USA

ABSTRACT

This chapter discusses the false rates of Bloom filters in a distributed environment. A Bloom filter (BF) is a space-efficient data structure to support probabilistic membership query. In distributed systems, a Bloom filter is often used to summarize local services or objects and this Bloom filter is replicated to remote hosts. This allows remote hosts to perform fast membership query without contacting the original host. However, when the services or objects are changed, the remote Bloom replica may become stale. This chapter analyzes the impact of staleness on the false positive and false negative for membership queries on a Bloom filter replica. An efficient update control mechanism is then proposed based on the analytical results to minimize the updating overhead. This chapter validates the analytical models and the update control mechanism through simulation experiments.

INTRODUCTION TO BLOOM FILTERS

A standard Bloom filter (BF) (Bloom, 1970) is a lossy but space-efficient data structure to support membership queries within a constant delay. As shown in Figure 1, a BF includes $k$ independent random hash functions and a vector $B$ of a length of $m$ bits. It is assumed that the BF represents a finite set $S = \{x_1, x_2, \ldots, x_n\}$ of $n$ elements from a universe $U$. The hash functions $h_i(x)$, $1 \leq i \leq k$, map the universe $U$ to the bit address space $[1, m]$, shown as follows,

$$H(x) = \{h_i(x) \mid 1 \leq h_i(x) \leq m \text{ for } 1 \leq i \leq k\}$$ (1)
Definition 1. For all \( x \in U \), \( B[H(x)] = \{B[h_i(x)] | 1 \leq i \leq k\} \).

This notation facilitates the description of operations on the subset of \( B \) addressed by the hash functions. For example, \( B[H(x)] = 1 \) represents the condition in which all the bits in \( B \) at the positions of \( h_1(x), \ldots, h_k(x) \) are 1. “Setting \( B[H(x)] \)” means that the bits at these positions in \( B \) are set to 1.

Representing the set \( S \) using a BF \( B \) is fast and simple. Initially, all the bits in \( B \) are set to 0. Then for each \( x \in S \), an operation of setting \( B[H(x)] \) is performed. Given an element \( x \), to check whether \( x \) is in \( S \), one only needs to test whether \( B[H(x)] = 1 \). If no, then \( x \) is not a member of \( S \); If yes, \( x \) is conjectured to be in \( S \). Figure 1 shows the results after the element \( x \) is inserted into the Bloom filter.

A standard BF has two well-known properties that are described by the following two theorems.

**Theorem 1. Zero false negative**

For \( \forall x \in U \), if \( \exists i, B[h_i(x)] \neq 1 \), then \( x \notin S \).

For a static set \( S \) whose elements are not dynamically deleted, the bit vector indexed by those hash functions always never returns a false negative. The proof is easy and is not given in this chapter.

**Theorem 2. Possible false positive**

For \( \forall x \in U \), if \( B[H(x)] = 1 \), then there is a small probability \( f^+ \) that \( x \notin S \). This probability is called the false positive rate and \( f^+ \approx (1 - e^{-kn})^k \). Given a specific ratio of \( m/n \), \( f^+ \) is minimized when \( k = (m/n)\ln 2 \) and \( f^+_{\text{min}} \approx (0.6185)^{m/n} \).

**Proof:** The proof is based on the mathematical model proposed in (James, 1983; McIlroy, 1982). Detailed proof can be found in (Li et al., 2000; Michael, 2002). For the convenience of the reader, the proof is briefly presented here.

After inserting \( n \) elements into a BF, the probability that a bit is zero is given by:
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