Advances in the Quotient Space Theory and its Applications

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ABSTRACT

Quotient space theory (QST), a new granule computing tool dealing with imprecise, incomplete and uncertain knowledge, uses a triplet, including the universe, its structure and attributes, to describe a problem space or simply a space. As one of important theories of granular computing (GrC), QST is very helpful to the study of cognitive informatics (CI). This article summarizes the quotient space’s model and its main principle. Then some basic operations on quotient space are introduced, and the significant properties of the fuzzy quotient space family are elaborated. Finally the main applications of quotient space theory are discussed. [Article copies are available for purchase from InfoSci-on-Demand.com]

Keywords: Data Mining; Decision Tree; Fuzzy Set; Granule Computing; Quotient Space; Rough Set

INTRODUCTION

Cognitive Informatics (CI) is a transdisciplinary enquiry of the internal information processing mechanisms and processes of the brain and natural intelligence that draws together many fields, including modern informatics, computation, software engineering, artificial intelligence, cybernetics, neuropsychology, medical science, etc. (Wang, 2003a, b; Yao, 2006). Granular computing (GrC) which imitates the manner of human thinking and covers all the research of the theories, methodologies, technologies, and tools about granules is the foundation of artificial intelligence. It has been rapidly developed by the practical needs for problem solving (Zadeh, 1998; Yao & Zhong, 1999; Yao, 2000, 2005; Lin et al., 2002; Wang et al., 2003). It can be seen that GrC may offer a conceptual framework for study of CI (Lin, 1997; Zadeh, 1997; Zhang & Zhang, 2003; Yao, 2006; Zhao & Zhang, 2008).

Quotient space theory (QST), a new granule computing tool to deal with imprecise, incomplete and uncertain knowledge, was introduced by Ling Zhang and Bo Zhang in 1989 (Zhang & Zhang, 1989a, b, 1990a, b). It aims at studying the relationship of transform and that of dependency among the worlds with different grain-size. It combines different granularities with the concept of mathematical quotient set.
and the problem spaces with different grain sizes can be represented and analyzed hierarchically by a set of quotient spaces. While the essence of human cognition is that human can observe and analyze a problem at different grain sizes but also translate from one granule world to the others with no difficulty, back and forth freely. So the study of QST is naturally helpful to the development of CI.

Compared with other granular computing theories, QST has more powerful abilities of representation and absorption. The reason is that structure is introduced in its model. It can not only describe the elements in the universe and different structure relationship among the elements, but also define a lot of different attribute functions and operations, which makes it translating from one granule world to the others easily. It can absorb the methods of most theories which are relatively mature at the present time, such as decision tree (Zhang et al., 2004c; Zhang & Zhang, 1992), fuzzy set (Zhang & Zhang, 2003, 2005b, c), rough set (Zhao & Zhang, 2008; Zhang et al., 2004b; Zhang & Zhang, 2003), analysisisitus (Zhang & Zhang, 1992), evidence theory (Zhang & Zhang, 1992), probability theory (Zhang & Zhang, 1992), wavelet analysis (Zhang & Zhang, 2005a), etc.

This article summarizes the quotient space’s model and its main principles including false-preserving, true-preserving, weak false-preserving, weak true-preserving and quotient approaching. Then we introduce its basic operations on quotient space such as projection, combination, quotient operation, quotient restriction and quotient approaching. Besides, the significant properties of the fuzzy quotient space are elaborated. Finally we discuss its main applications in the areas like machine learning, biological sequence alignment, fuzzy control and communication countermeasure reconnaissance.

**THE MODEL OF QUOTIENT SPACE THEORY AND ITS MAIN FEATURES**

The quotient space theory combines the different granularities with the concept of mathematical quotient set and represents a problem by a triplet \((X, f, T)\), where \(X\) is the set of our discussing object, namely the universe; \(f(\bullet)\) is the attribute function of universe \(X\), and it may be multidimensional, \(f = (f_1, ..., f_n), f_i : X \rightarrow Y_i\), which can be divided into two classes, the condition attribute and the decision attribute, denoted by the set \(C\) and \(D\) respectively, then \(f = C \cup D\); \(T\) is the structure of universe \(X\), namely the interrelations of elements.

When we view the same universe \(X\) from a coarser grain size, that is, when we give an equivalence relation \(R\) on \(X\), we can get a corresponding quotient set denoted by \([X]\), where \([X] = \{[x]\} | [x] = \{y | x Ry\}, x \in X\}\). Then viewing \([X]\) as a new universe, we must have the corresponding coarse-grained space \(([X], [f], [T])\) called a quotient space of \((X, f, T)\). Similarly we can construct a quotient space of \((X, f, T)\) by taking a coarser grain size on \(T\) or \(f\).

**The Construction of Quotient Space**

**The Construction of Quotient Structure \([T]\)**

**Definition 2.1.** A problem space \(([X], [f], [T])\) is called a semi-order space if there exists a relation “<” among part of elements on \(X\) and satisfies: (1) if \(x < y\) and \(y < x\), then \(x = y\); (2) if \(x < y\) and \(y < z\), then \(x < z\). If the condition (1) does not hold, the relation < is called a pseudo-semi order relation.

**Definition 2.2.** Given a semi-order space \((X, T)\) and an equivalence relation \(R\) on \(X\). If the quotient space \(([X], [T])\) corresponding to \(R\), is also a semi-order space, then we say that \(R\) and \(T\) are compatible, or \(R\) is compatible, for short.
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