Chapter 17

Unifying Rough Set Analysis and Formal Concept Analysis Based on a Logic Approach to Granular Computing

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ABSTRACT

Granular computing is an emerging field of research that attempts to formalize and explore methods and heuristics for human problem solving with multiple levels of granularity and abstraction. A fundamental issue of granular computing is the construction, representation and utilization of granules and granular structures. Basic granules represent the basic pieces of knowledge. A granular structure reflects the connections between different pieces of knowledge. The main objective of this book chapter is to examine a logic approach to granular computing for combining rough set analysis and formal concept analysis. Following the classical interpretation of concepts that a concept consists of a pair of an extension and an intension, the authors interpret a granule as a pair containing a set of objects and a logic formula describing the granule. The building blocks of granular structures are basic granules representing elementary concepts or pieces of knowledge. They are treated as atomic formulas of a logic language. Different types of granular structures can be constructed by using logic connectives. Within this logic framework, this chapter shows that rough set analysis and formal concept analysis can be interpreted uniformly by using the proposed logic language. The two theories share high-level similarities, but differ in their choices of definable granules and granular structures. Algorithms and evaluation measures can be designed uniformly for both theories.

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INTRODUCTION

Cognitive science (Simon & Kaplan, 1989) and cognitive informatics (Wang, 2003a, 2003b) study information and knowledge processing in the abstract, in the brain, and in machines. Some of the salient features of human intelligence and problem solving are the conceptualization of a problem at multiple levels of abstraction, the representation of information and knowledge with different-sized granules, the choice of a suitable level of granularity, and the switching of views and granularity in response to changes in environments. An emerging field of study known as granular computing aims at formalizing and exploring these features (Bargiela & Pedrycz, 2002; Yao, 2004c, 2006b, 2007a; Zadeh, 1997). The results from granular computing may shed new light on the study of cognitive informatics (Wang, 2003a, 2003b, 2007a, 2007b; Yao, 2006a).

A central notion of granular computing is multilevel granular structures consisting of a family of interrelated and interacting granules. Granular computing can be considered as an umbrella term covering topics that concern granularity and has been studied either implicitly or explicitly in many fields. It focuses on problem solving by describing and representing a problem and its solution in various levels of granularity so that one can focus on things that serve a specific interest and ignore unimportant and irrelevant details. Granular computing makes use of knowledge structures and hence has a significant impact on the study of human intelligence and the design of intelligent systems.

Granular computing can be studied based on a triarchic model consisting of the philosophy of structured thinking, the methodology of structured problem solving, and the computation of structured information processing (Yao, 2001b, 2004d, 2006b, 2007a). Many concrete models of granular computing have been proposed and studied. The main objective of the book chapter is to make further contribution along this line by investigating a logic approach to granular computing.

We introduce a logic language L to study granular computing in a logic setting. The language is an extension of the decision logic language used in rough set theory (Pawlak, 1991). Instead of expressing atomic formulas by a particular concrete type of conditions, we treat them as abstract notions to be interpreted in different applications. This flexibility enables us to describe granules in different applications. The language is interpreted in the Tarski’s style through the notion of a model and satisfiability (Demri & Orlowska, 1997; Pawlak, 1991; Pawlak & Skowron, 2007; Yao, 2001b; Yao & Liau, 2002). The model is defined as a pair consisting of a domain and knowledge about the domain. The meaning of a formula is given by a set of objects satisfying the formula. Like the representation of a concept by a pair of intension and extension, a granule is interpreted as a pair of a set of objects of the domain and a formula of the language L. Thus, we can study granular structures in both a set-theoretic setting and a logic setting. The basic granules are represented by atomic formulas. An object satisfies a formula if the object has the properties as specified by the formula.

Rough set analysis and formal concept analysis are two concrete models of granular computing for knowledge representation and data analysis (Nguyen, Skowron & Stepaniuk, 2001; Pawlak, 1991; Pawlak & Skowron, 2007; Wille, 1982, 1992; Yao, 2004a). Rough set analysis studies the object-attribute relationships in an information table and formal concept analysis studies these relationships in single-valued and many-valued formal contexts. With the introduced language, the two theories can be interpreted in a unified way. On one hand, the two theories share high-level similarities in their treatments of granular structures. On the other hand, they use different atomic formulas, definable granules and granular structures formed by definable granules. The unified study of the two theories not only