Chapter 20
A Relative Fractal Dimension Spectrum for a Perceptual Complexity Measure

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ABSTRACT

This article presents a derivation of a new relative fractal dimension spectrum, DRq, to measure the dis-similarity between two finite probability distributions originating from various signals. This measure is an extension of the Kullback-Leibler (KL) distance and the Rényi fractal dimension spectrum, Dq. Like the KL distance, DRq determines the dissimilarity between two probability distributions X and Y of the same size, but does it at different scales, while the scalar KL distance is a single-scale measure. Like the Rényi fractal dimension spectrum, the DRq is also a bounded vectorial measure obtained at different scales and for different moment orders, q. However, unlike the Dq, all the elements of the new DRq become zero when X and Y are the same. Experimental results show that this objective measure is consistent with the subjective mean-opinion-score (MOS) when evaluating the perceptual quality of images reconstructed after their compression. Thus, it could also be used in other areas of cognitive informatics.

INTRODUCTION

This article is related to measuring the quality of various multimedia materials used in perception, cognition and evolutionary learning processes. The multimedia materials may include temporal signals such as sound, speech, music, biomedical and telemetry signals, as well as spatial signals such as still
images, and spatio-temporal signals such as animation and video. A comprehensive review of the scope of multimedia storage and transmission, as well as quality metrics is presented (Kinsner, 2002). Most of multimedia original materials are altered (compressed, or enhanced, or watermarked) either to fit the available storage or bandwidth during their transmission, or to enhance perception of the materials, or to protect the original material from alterations and copying. Since the signals may also be contaminated by noise during different stages of their processing and transmission, various denoising techniques must be used to minimize the noise, without affecting the signal itself (Kinsner, 2002). Different classes of coloured and fractal noise are described (Kinsner, 1994, June 15). A review of approaches to distinguish broadband signals and noise from chaos was provided (Kinsner, 2003). Very often, multimedia compression is lossy in that the signals are altered with respect not only to their redundancy, but also to their perceptual and cognitive relevancy. Since the signals are presented to humans (rather than machines), cognitive processes must be considered in the development of suitable quality metrics. Energy-based metrics are not suitable for such cognitive processes (Kinsner, 2004). A very fundamental class of metrics based on entropy was described (Kinsner, 2004), with a discussion on its usefulness and limitations in the area of cognitive informatics (CI) (Wang, 2002) and autonomic computing (Kinsner, 2005a). An extension from the single-scale entropy-based metrics to multiscale metrics through a unified approach to fractal dimensions was presented in (Kinsner, 2005b). The main realization presented in that article was that not all fractal dimensions are the same, and that the morphological (projective) dimensions are just a subset of the entropy-based dimensions. That article also stressed the importance of moving away from scalar to vectorial, multiscale, and bounded measures in CI, which is a requirement satisfied by the Rényi fractal dimension spectrum.

This article presents a fundamental extension in the form of a new \textit{relative fractal dimension spectrum} that is not only multiscale, vectorial and bounded, but also produces a zero vector if the distributions of the source $X$ and its reconstruction $Y$ are the same. Experimental results obtained by the authors indicate that this quality measure is the best perceptual quality metrics devised so far (Dansereau, 2001; Dansereau & Kinsner, 2001; Kinsner & Dansereau, 2006).

Other practical applications of the new relative measure include its use in: (i) the fine tuning of algorithms to compress or enhance signals, and (ii) the improved feature extraction for characterization and classification by neural networks.

\textbf{BACKGROUND ON FRACTAL MEASURES}

\textbf{Single Fractal Dimension Measures}

In one form or another, all fractal dimension measures determine the critical exponent that stabilizes a power-law relation of the following form:

\begin{equation}
N_{s_k} : s_k^{D_H} \text{ for } s_k \rightarrow 0, \ k \rightarrow \infty
\end{equation}

where $N_{s_k}$ is a measure of the complexity of the object or data set (e.g., a simple count of a covering (Kinsner, 2005b), or a functional measure on the covering) at the $k$th \textit{scale} of the measurements, $s_k$. This scale $s_k$ is related to the \textit{reduction factor} $r_k$ of a volume element (vel for short) at each $k$th covering through $s_k = (1/r_k)$ (Kinsner, 2005b). For example, if the reduction factor is 1/3, the scale is 3. The sign ~