On the Ordering Property and Law of Importation in Fuzzy Logic

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ABSTRACT

In this paper, the authors investigate the ordering property (OP), \( x \leq y \iff I(x, y) = 1 \), together with the general form of the law of importation (LI), i.e., \( I(T(x, y), z) = I(x, I(y, z)) \), where \( T \) is a t-norm and \( I \) is a fuzzy implication for the four main classes of fuzzy implications. The authors give necessary and sufficient conditions under which both (OP) and (LI) holds for S-, R-implications and some specific families of QL-, D-implications. Following this, the paper proposes the sufficient condition under which the equivalence between CRI and triple I method for FMP can be established. Moreover, this conclusion can be viewed as a unified triple I method, a generalized form of the known results proposed by Wang and Pei.

Keywords: Compositional Rule of Inference, Full Implication Triple I Method, Fuzzy Implications, Law of Importation, Ordering Property

INTRODUCTION

A lot of tautologies valid in classical logic are generally not true when they are translated to fuzzy logic, using operators like t-norms, t-conorms and strong negations to perform conjunctions, disjunctions and negations, respectively. Lately, there have been a large amount of efforts to discuss and explore for which of these operators, a classical tautology remains true (Trillas & Alsina, 2002; Balasubramaniam & Rao, 2004; Combs & Andrews, 1998).

One of the classical logic tautologies that have attracted maximum attention from researchers is the law of importation:

\[
(x \land y) \rightarrow z \equiv (x \rightarrow (y \rightarrow z))
\]

the general form of it is:

\[
I(T(x, y), z) = I(x, I(y, z)) \quad \text{(LI)}
\]

In the framework of fuzzy logic, (LI) has been studied in isolation (Jayaram, 2008) as well.

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as with some related properties, for instance, Baczyński (2001) and Bouchon-Meunier and Kreinovich (1996) have studied (LI) with the general form of the following distributive property:

\[ x \rightarrow (y \land z) \equiv (x \rightarrow y) \land (x \rightarrow z). \]

However, the law of importation has not been studied in conjunction with the ordering property:

\[ x \leq y \iff I(x, y) = 1 \quad (OP) \]

which means the implication \( I \) defines an ordering on propositions (Dubois & Prade, 1991). In this paper, we investigate the conditions under which the four most established and well-studied classes of fuzzy implications, \( S- \), \( R- \), \( QL- \) and \( D- \) implications, satisfy both (OP) and (LI) and explore their potential applications in the field of approximate reasoning along the following lines.

The full implication triple I method, or simply the triple I method, proposed by Wang (1999) is one of the most established methods of approximate reasoning. This method may bring approximate reasoning within the framework of logical semantic implication, and it may be considered as a reasonable alternative or complement for the compositional rule of inference (CRI) proposed by Zadeh (1973). In the past few years, there were a lot of discussion centred on the unified form of triple I algorithm. However, these algorithms were constructed only for some special implications. Both Wang and Fu (2005) and Pei (2008) designed several algorithms for the uniform of the triple I method, but mostly based on residuated implications. Song, Feng, and Lee (2002) investigated the triple I method by using Zadeh’s implication operator \( R_z \), and obtained the computational formula for fuzzy modus ponens (FMP). Moreover, their proofs tend to be very complicated.

We propose a further generalized algorithm for the uniform of the triple I method by investigating when the inference obtained from the CRI and triple I method become equivalent. Toward this end, we give some sufficient conditions on the operators employed in the inference, conditions that highlight the significant role played by the ordering property and the law of importation. Contrast with the complicated proofs in the literatures we mentioned before, the proof of our conclusion is greatly simplified.

The paper is organized as follows. First, we review some basic notations and definitions of fuzzy logic connectives, fuzzy implications, including \( S- \), \( R- \), \( QL- \) and \( D- \) implications and some preliminary properties regarding these fuzzy implications. We then prove the sufficient and necessary conditions for an \( S- \) implication generated by any \( t- \) conorm and a strong fuzzy negation, an \( R- \) implication generated by a left-continuous \( t- \) norm, \( QL- \) and \( D- \) implications generated by continuous \( t- \) conorms, any \( t- \) norm and strong fuzzy negations to satisfy both (OP) and (LI). We also show the sufficient conditions under which the equivalence between the inference in the CRI and triple I method can be obtained. Furthermore, we propose the generalized algorithm for the uniform of the triple I method and highlight its advantages.

**Preliminaries**

To make this paper self-contained, we recall here some definitions and results that we will specially use in the rest of this paper.

**Basic Fuzzy Logic Connectives**

In this part, we introduce basic notations of fuzzy logic connectives used in the text and we briefly mention some of the concepts and results employed in the rest of the work.

**Definition 1** Let \( \phi : [0, 1] \rightarrow [0, 1] \) be an increasing bijection. If \( f : [0, 1]^n \rightarrow [0, 1] \), \( n \in \mathbb{N} \), then the \( \phi \) - conjugate of \( f \) is given by
Incorporation of Preferences in an Evolutionary Algorithm Using an Outranking Relation: The EvABOR Approach
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