Preinvexity and Semi-Continuity of Fuzzy Mappings

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ABSTRACT

In this paper, the relationships between semicontinuity and preinvexity of fuzzy mappings are investigated. The concepts of intermediate-point preinvex fuzzy mappings and weakly preinvex fuzzy mappings are first introduced, and the authors establish a characterization for a weakly preinvex fuzzy mapping in terms of its epigraph. In this context, a characterization of a preinvex fuzzy mapping in terms of its relationship with intermediate-point preinvexity and weakly preinvexity, respectively, is obtained.

Keywords: Convex Fuzzy Mapping, Convexity, Fuzzy Mapping, Preinvexity, Semicontinuity

1. INTRODUCTION

Convexity and semicontinuity of fuzzy mappings play a central role in the theories of fuzzy optimization and fuzzy nonlinear programming. Various investigators (Furukawa, 1998; Nanda & Kar, 1992; Syau, 1999; Wang & Wu, 2003) studied and developed criteria for convex fuzzy mappings. The concept of upper and lower semicontinuity of fuzzy mappings based on the Hausdorff separation was introduced by Diamond and Kloeden (1994). Recently, Bao and Wu (2006) introduced new concepts of upper and lower semicontinuous fuzzy mappings through the “fuzz-max” order on fuzzy numbers in ways that parallel closely to the corresponding definitions for upper and lower semicontinuous real-valued functions. They also obtained criteria for convex fuzzy mappings under upper and lower semicontinuity conditions. In an earlier paper (Syau, Sugianto, & Lee, 2008), we redefined the upper and lower semicontinuity of fuzzy mappings of Bao and Wu (2006) by using the concept of parameterized triples of fuzzy numbers.

Noor (1994) extended the concept of preinvexity to fuzzy mappings. It was proved in Noor (1994) and our earlier paper (Syau, 1999) that preinvex fuzzy mappings are more general than convex fuzzy mappings, and that preinvexity can be substituted for convexity in many results in mathematical fuzzy programming involving convex fuzzy mappings.

Motivated by earlier research works, and by the importance of preinvexity and semicontinuity of fuzzy mappings, the relationships between semicontinuity and preinvexity of fuzzy mappings are investigated in this study. We introduce the concepts of intermediate-point...
preinvex fuzzy mappings and weakly preinvex fuzzy mappings. We also give a characterization for a weakly preinvex fuzzy mapping in terms of its epigraph. A characterization of a preinvex fuzzy mapping in terms of its relationship with intermediate-point preinvexity and weakly preinvexity, respectively, is obtained.

2. PRELIMINARIES

In this section, for convenience, several definitions and results without proof from (Bao & Wu, 2006; Mohan & Neogy, 1995; Noor, 1994; Syau et al., 2008; Weir & Jeyakumar, 1988; Weir & Mond, 1988; Yang, 2001) are summarized below.

Let \( \mathbb{R}^n \) denote the \( n \)-dimensional Euclidean space, and let \( S \) be a nonempty subset of \( \mathbb{R}^n \). For any \( x \in \mathbb{R}^n \) and \( \delta > 0 \), let

\[
B(x, \delta) = \{ y \in \mathbb{R}^n : \| y - x \| < \delta \},
\]

where \( \| \cdot \| \) being the Euclidean norm on \( \mathbb{R}^n \).

First, we recall the definitions of upper and lower semicontinuous real-valued functions.

**Definition 2.1.** A real-valued function \( f : S \to \mathbb{R}^i \) is said to be

1. Upper semicontinuous at \( x_0 \in S \) if \( \forall \varepsilon > 0 \), there exists a \( \delta = \delta(x_0, \varepsilon) > 0 \) such that

\[
f(x) < f(x_0) + \varepsilon \text{ whenever } x \in S \cap B(x_0, \delta).
\]

2. Lower semicontinuous at \( x_0 \in S \) if \( \forall \varepsilon > 0 \), there exists a \( \delta = \delta(x_0, \varepsilon) > 0 \) such that

\[
f(x) > f(x_0) - \varepsilon \text{ whenever } x \in S \cap B(x_0, \delta).
\]

\( f \) is lower semicontinuous on \( S \) if it is lower semicontinuous at each point of \( S \).

The support, \( \text{supp}(\mu) \), of a fuzzy set \( \mu : \mathbb{R}^n \to I = [0, 1] \) is defined as

\[
\text{supp}(\mu) = \{ x \in \mathbb{R}^n : \mu(x) > 0 \}.
\]

A fuzzy set \( \mu : \mathbb{R}^n \to I \) is normal if \( [\mu]_0 \neq \emptyset \). A fuzzy number we treat in this study is a fuzzy set \( \mu : \mathbb{R}^i \to I \) which is normal, has bounded support, and is upper semicontinuous and quasiconcave as a function on its support.

Denote by \( F \) the set of all fuzzy numbers. In this paper, we consider mappings \( F \) from a nonempty subset of \( \mathbb{R}^n \) into \( F \). We call such a mapping a fuzzy mapping. It is clear that each \( r \in \mathbb{R}^i \) can be considered as a fuzzy number \( \tilde{r} \) defined by

\[
\tilde{r}(t) = 1, \text{if } t=r; 0, \text{if } t \neq r.
\]

And hence, each real-valued function can be considered as a fuzzy mapping.

Let \( \alpha \in I \). The \( \alpha \)-level set of a fuzzy set \( \mu : \mathbb{R}^n \to I \), denoted by \( [\mu]_\alpha \), is defined as

\[
[\mu]_\alpha = \{ x \in \mathbb{R}^n : \mu(x) \geq \alpha \},
\]

if \( 0 < \alpha \leq 1 \); \( \text{cl}(\text{supp}(\mu)), \text{if } \alpha = 0 \),

where \( \text{cl}(\text{supp}(\mu)) \) denotes the closure of \( \text{supp}(\mu) \).

It can be easily verified that a fuzzy set \( \mu : \mathbb{R}^i \to I \) is a fuzzy number if and only if

1. \( [\mu]_0 \) is a closed and bounded interval for each \( \alpha \in I \), and (ii) \( [\mu]_1 = \emptyset \). Thus we can identify a fuzzy number \( \mu \) with the parameterized triples

\[
\{(a(\alpha), b(\alpha), \alpha) : \alpha \in I\},
\]

\( a(\alpha) \) and \( b(\alpha) \) being the left and right end points of the interval, respectively.

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