Chapter 19

Pattern Formation Controlled by External Forcing in a Spatial Harvesting Predator–Prey Model

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ABSTRACT

Predator–prey models in ecology serve a variety of purposes, which range from illustrating a scientific concept to representing a complex natural phenomenon. Due to the complexity and variability of the environment, the dynamic behavior obtained from existing predator–prey models often deviates from reality. Many factors remain to be considered, such as external forcing, harvesting and so on. In this chapter, we study a spatial version of the Ivlev-type predator-prey model that includes reaction-diffusion, external periodic forcing, and constant harvesting rate on prey. Using this model, we study how external periodic forcing affects the stability of predator-prey coexistence equilibrium. The results of spatial pattern analysis of the Ivlev-type predator-prey model with zero-flux boundary conditions, based on the Euler method and via numerical simulations in MATLAB, show that the model generates rich dynamics. Our results reveal that modeling by reaction-diffusion equations with external periodic forcing and nonzero constant prey harvesting could be used to make general predictions regarding predator-prey equilibrium, which may be used to guide management practice, and to provide a basis for the development of statistical tools and testable hypotheses.

INTRODUCTION

Ecology has grown to be an enormous field of science in recent years (Murray, 2003; Wang et al., 2007). The topics of research include the study of the interrelationship between species and their environment, predator-prey and competition interactions, renewable resource management, evolution of pesticide resistant strains, ecological and genetic engineering in pest control, multi-species societies, plant-herbivore systems, and so on.

Predation, a complex natural phenomenon, exists widely in the world, e.g., in the sea, on the plain, in the forest, in the desert, and so on.
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Since the pioneering work of Lotka and Volterra (reviewed in Kendall, 2001), the predator-prey model has been widely studied. The predator-prey model is a type of “pursuit and evasion” system in which the prey tries to evade the predator and the predator tries to catch the prey when they interact (Murray, 2003). Pursuit means that the predator tries to shorten the spatial distance between the predator and the prey, whereas evasion means that the prey tries to widen this spatial distance. The predator-prey model is a mathematical model approximating such real-world phenomenon. Predator-prey model will likely continue to be widely researched due to its universality and importance (Berryman, 1992; Kuang & Beretta, 1998).

A typical spatial extended model, reaction-diffusion model, involves not only time but also space and consists of several species which react with each other and diffuse within the spatial domain. It normally uses a pair of partial differential equations to represent the time course of reacting and diffusing processes. In the spatial extended predator-prey model, the reaction represents the event when the predator catches and consumes the prey, and the diffusion is a spatial process and describes the predator’s “pursuit” and the prey’s “evasion”. The reaction-diffusion model describes the evolution of the predator and the prey as a function of time (Zhang, 2008).

Harvest is the process of gathering mature crops from the fields. The harvest marks the end of the growing season, or the growing cycle for a particular crop. Harvesting, in general usage, includes an immediate post-harvest handling, such as all of the actions taken immediately after removing the crop—cooling, sorting, cleaning, packing—up to the point of further on-farm processing, or shipping to the wholesale or consumer market. The exploitation of biological resources and the harvesting are commonly practiced in fishery, forestry, and wildlife management. Concerning the conservation, for the long-term benefits of humanity, there is a wide-range of interest in the use of bioeconomic modeling of proper harvesting to gain insight on the scientific management of renewable resources such as fisheries and forestry (Xiao et al., 2006). Harvesting is also an important factor affecting the predator-prey system.

For a spatial extended predator-prey model, it is well known that the internal source is the dynamics of the individuals of the model and the external source is the variability of the environment. These two sources act together in deciding the evolution of the model (Zhou & Kurths, 2005). Some of the variability is periodic, such as temperature, water, food supply of the prey, and mating habits. It is necessary and important to consider models with periodic ecological parameters or perturbations. These periodic factors are regarded as the external periodic forcing in the predator-prey model. The external forcing can affect the populations of the predator and the prey, which would go extinct in a deterministic environment (Huang & Jing, 2009; Liu et al., 2008).

In this chapter, we consider the Ivlev-type predation-diffusion model with constant harvest, which has not been widely studied. Ivlev (1961) studied the spatiotemporal complexity of a predator-prey model with Ivlev-type functional response (Ivlev, 1961). Functional response, that is, the prey consumption rate by an average single predator, increases with the prey consumption rate. Functional response can be influenced by the predator density. Functional response of the predator to the prey density refers to the change in the density of prey attached per unit time per predator as the prey density changes. Wang et al, (2010) focused on the following diffusive model:

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\begin{align*}
\frac{\partial N}{\partial t} &= N(1 - N) - P(1 - e^{-\gamma N}) + d_1 \nabla^2 N, \\
\frac{\partial P}{\partial t} &= \varepsilon P(1 - e^{-\gamma N}) - \beta P + d_2 \nabla^2 P.
\end{align*}
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(1)