Chapter 5
Optimality-Oriented Stabilization for Recurrent Neural Networks

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ABSTRACT
This chapter presents an approach of how optimality-oriented stabilization is achieved for recurrent neural networks, which includes both the input-to-state stabilization for deterministic recurrent neural networks and the noise-to-state stabilization for stochastic recurrent neural networks. Owing to the difficulty in solving the Hamilton-Jacobi equation for nonlinear systems, optimal regulation seems to be an unachievable goal in control design for recurrent neural networks. However, a methodology proposed in this chapter solves the problem and obtains optimal stabilization by using the knowledge of Lyapunov technique, inverse optimality, and differential game theory. Numerical examples demonstrate the effectiveness of the proposed design.

INTRODUCTION
Inspired by biological neuronal systems, artificial neural networks have demonstrated superior characteristics of learning, adaptation, classification and function-approximation. They have been successfully applied to many areas. However, conventional neural networks are feed-forward networks that move the information in only one direction, forward, from the inputs to the outputs. There are no cycles or loops in the network and no feedback connections are present. Therefore, there are some drawbacks in using those networks. Since the Hopfield’s paper (Hopfield, 1984), recurrent neural networks, also called dynamic neural networks, have emerged to be a highly effective methodology. Because these networks have feedback connections that represent dynamic memory, which make them more suitable for modeling biological intelligence than purely feed-forward neural networks. In addition, within the structure of recurrent neural networks, they have bi-directional data flow. Therefore, they have been used to solve various difficult problems in many
scientific areas, such as pattern recognition, operational analysis, system identification and control, and content-addressable memory. Moreover, they are easy for VLSI implementation. Motivated by the successful applications of recurrent neural networks, theoretical studies on both stability and controllability of recurrent neural networks have emerged in the last few years, see for example, (Arik, 2000; Ensari & Arik, 2005; Hu & Wang, 2002; Kulawski & Brdys, 2000; Liu, Torres, Patel, & Wang, 2008; Liu, Wang, & Schurz, 2009; Liu & Wang, 2007; Sanchez & Perez, 2003), and references therein. The latest research results, obtained in (Liu, Torres, Patel, & Wang, 2008; Liu, Wang, & Schurz, 2009; Liu & Wang, 2007), solve both the problem of the input-to-state stabilization for deterministic recurrent neural networks and the problem of the noise-to-state stabilization for stochastic recurrent neural networks.

Based on the results in (Liu, Torres, Patel, & Wang, 2008; Liu, Wang, & Schurz, 2009; Liu & Wang, 2007), this chapter provides a rigorous theoretical foundation to the theory and algorithms of optimality-oriented stabilization for recurrent neural networks, which includes both input-to-state stabilization for deterministic recurrent neural networks and noise-to-state stabilization for stochastic recurrent neural networks. In order to achieve the aforementioned goals, the rest of the chapter is organized as follows. Part A presents the theoretical results of input-to-state stabilization for deterministic recurrent neural networks, together with two numerical examples to show the effectiveness. Part B illustrates an approach to achieve noise-to-state stabilization in probability for stochastic recurrent neural networks driven by noise of unknown covariance. In addition, two numerical examples are given. Finally, the main conclusions are reported.

PART-A: INPUT-TO-STATE STABILIZATION FOR DETERMINISTIC RECURRENT NEURAL NETWORKS

This part considers the design of input-to-state stabilization for deterministic recurrent neural networks. This approach is developed by using Lyapunov technique, inverse optimality, and Hamilton-Jacobi-Bellman (HJB) equation. Depending on the dimensions of state and input, two optimal control laws are constructed in order to achieve input-to-state stabilization for the networks. Furthermore, the proposed designs achieve global asymptotic stability and global inverse optimality with respect to some meaningful cost functional. Two numerical examples demonstrate the performance of the approach. The content of this part is based on (Liu, Torres, Patel, & Wang, 2008) and (Liu & Wang, 2007).

Problem Formulation and Mathematical Preliminaries

Based on the formulation of recurrent neural networks, we consider the following deterministic recurrent neural networks.

\[
\dot{x}_i(t) = -\lambda x_i(t) + \sum_{k=1}^{n} w_{ik}^s s(x_k) + \sum_{j=1}^{m} w_{ij}^u u_j
\]

(1)

where \(i = 1, 2, \ldots, n\). Mathematically, this can be described by the following compact form

\[
\dot{x} = -Ax + W_s S(x) + W_u u
\]

(2)

where \(x \in \mathbb{R}^n\) is the state of recurrent neural networks, \(u \in \mathbb{R}^m\) is the input, usually \(m \neq n\), \(A = \text{diag}(-\lambda, \ldots, -\lambda) = -\lambda I \in \mathbb{R}^{n \times n}\) and \(\lambda > 0\), \(S(x) = [s(x_1), \ldots, s(x_n)]^T \in \mathbb{R}^n\) is a nonlinear vector-valued activation function with \(s(\cdot) = \ldots\)
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