Chapter 6
Design of Globally Robust Control for Biologically-Inspired Noisy Recurrent Neural Networks

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ABSTRACT
This chapter presents a theoretical design of how a global robust control is achieved in a class of noisy recurrent neural networks which is a promising method for modeling the behavior of biological motor-sensor systems. The approach is developed by using differential minimax game, inverse optimality, Lyapunov technique, and the Hamilton-Jacobi-Isaacs equation. In order to implement the theory of differential games into neural networks, we consider the vector of external inputs as a player and the vector of internal noises (or disturbances or modeling errors) as an opposing player. The proposed design achieves global inverse optimality with respect to some meaningful cost functional, global disturbance attenuation, as well as global asymptotic stability provided no disturbance. Finally, numerical examples are used to demonstrate the effectiveness of the proposed design.

INTRODUCTION
Recurrent neural networks have become one of the most promising methodologies to solve various difficult problems in different scientific areas, such as system identification and control, pattern recognition, image processing, modeling biological sensor-motor systems, etc. Therefore, theoretical studies on both stability and controllability of recurrent neural networks have received much attention during the last several years, see for example, (Arik, 2000; Ensari & Arik, 2005; Hu & Wang, 2002; Kulawski & Brdys, 2000; Liu, Torres, Patel, & Wang, 2008; Marco, Forti, & Tesi, 2004; Sanchez & Perez, 2003; Sontag & Qiao, 1999; Sontag, 1989), and references therein. However, these studies primarily focused on these mathematical models, which do not consider the noise process that is fraught with signal transmission particularly in biological systems.

On the other hand, Haykin (Haykin, 1999) indicated that the synaptic transmission is a noise
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...process in real nervous systems. Therefore, in order to build mathematical specifications for neural networks, which will be used as models of intelligence in the brain, and as highly effective artificial intelligent systems implemented in engineering and computer science, we must consider the noise or disturbance environment (Werbos, 2009). Hence, it is important to analytically explore the characteristics of stabilization and controllability for recurrent neural networks under the influence of noise or disturbance.

Based on our research (Liu, Shih, & Wang, 2009), this chapter provides a groundbreaking nonlinear $H_\infty$ optimal control for deterministic noisy recurrent neural networks to achieve a prescribed level of noise attenuation with stability margins. It is constructed by using differential minimax game, inverse optimality, Lyapunov technique, and the Hamilton-Jacobi-Isaacs equation. In order to realize a minimax equilibrium of differential game, we considered the vector of external inputs as a player and the vector of internal noises (or disturbances or modeling errors) as an opposing player. To complete the aforementioned goals, the rest of the chapter is organized as follows. The following section introduces the problem formulation. Next section shows the main results of a nonlinear $H_\infty$ control design for noisy recurrent neural networks. After that, two numerical examples are given to demonstrate the effectiveness of the proposed design. Finally, the main conclusions are reported.

**PROBLEM FORMULATION**

Consider the class of recurrent neural networks described by the following differential equations

$$\dot{x} = -Ax + W_1 S(x) + W_2 u + d$$

where $x \in \mathbb{R}^n$ is the state of recurrent neural network, $u \in \mathbb{R}^m$ is the input, usually $m \neq n$, $A \in \mathbb{R}^{n \times n}$ is a matrix that represents the neuron self-inhibitions, $S(x) = [s(x_1), \ldots, s(x_n)]^T \in \mathbb{R}^n$ is a vector function and its component $s(x)$ is a sigmoidal function that models the nonlinear input-output activations of the neurons, $W_1 \in \mathbb{R}^{n \times n}$, $W_2 \in \mathbb{R}^{n \times m}$ are weight matrices. Model (1) encompasses a large class of neural networks, such as the popular Hopfield neural networks, the paradigm of cellular neural networks, and many other recurrent neural network models frequently used in the literature.

In this chapter, we are interested in dealing with recurrent neural networks under the influence of noise or disturbance. Therefore, the model of noisy recurrent neural networks in the deterministic form can be considered as

$$\dot{x} = -Ax + W_1 S(x) + W_2 u + d$$

where $d \in \mathbb{R}^n$ is an unknown $n$-dimensional internal noise or disturbance or modeling error.

The design objective is to attenuate the noise to a prescribed level with stability margins. This can be achieved by implementing differential minimax game with the consideration of the vector of external inputs as a player and the vector of internal noises as an opposing player, which is equivalent to develop a nonlinear $H_\infty$ optimal control for the system (2). The main difficulty in obtaining the solution is in finding the solution of a Hamilton-Jacobi-Isaacs (HJI) equation. When the system dynamics are nonlinear, solving this HJI equation has presented a great challenge. However, the knowledge of inverse optimality allows us to find an alternative way to solve the problem and obtain optimal feedback controllers. The design procedure will be illustrated in the following section.
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