Chapter 15

Towards Arithmetical Chips in Sub-Excitable Media: Cellular Automaton Models

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ABSTRACT

We discuss a theoretical design of an arithmetical chip built on an excitable medium substrate. The chip is simulated in a two-dimensional three-state cellular automaton with eight-cell neighborhoods. Every resting cell is excited if it has exactly two excited neighbors, the excited cells takes refractory state unconditionally. A transition from refractory back to resting state also happens irrelevantly to a state of the cell neighborhood. The design is based on principles of collision-based computing. Boolean logic values are encoded by traveling localizations, or particles. Logical gates are realized in collisions between the particles. Detailed blue prints of collision-based adders and multipliers presented in the article pave the way to future laboratory experimental prototypes of general-purpose chemical computers.

INTRODUCTION

Excitable chemical media are amongst most promising ‘wet’ computing devices capable for solving a wide ranging of tasks from optimization, computational geometry, image processing and robot control. The excitable media can also implement functionally complete sets of logical gates thus qualifying as (logically) universal computing systems. See theoretical background and details of laboratory implementations of reaction-diffusion computers in (Adamatzky, De Lacy Costello, Asai, 2005).

So far all experimental laboratory prototypes of reaction-diffusion chemical computers are in fact specialized, or task oriented purposed processors. They are capable for solution of only one problem or family of problems. When logical universality is concerned the experimental implementations do usually deal with one or two logical gates, just as a matter of demonstration (Adamatzky, 2004; De Lacy Costello and Adamatzky, 2005; Adamatzky and De Lacy Costello, 2007; Toth et al, 2008). To move from abstract computational
uni versatility to general-purpose machine we must 
demonstrate how we can cascade simple logical 
gates in more complicated circuits able to execute 
sensible computational tasks. An implementation 
of arithmetical chip in reaction-diffusion medium 
would be enough to convince laymen that excit 
able chemical computing devices are not just a 
matter of curiosity but viable candidates for a role 
of future non-silicon computers.

We envisage that novel arithmetic chip, to 
be built in an excitable medium, will be based 
on principles of collision-based computing. The 
proposed chips will be based on logical schemes 
of computation in Conway’s Game-of-Life (Ber 
lekamp, Conway, Guy, 1982), Fredkin-Toffoli’s 
conservative logic (Fredkin and Toffoli, 1982) and 
Margolus’s physics of computation (Margolus, 
1984). In collision-based computing (Adamatzky, 
2003), quanta of information are represented by 
compact patterns traveling in an ‘empty’ space and 
performing computation by colliding with each 
other. The absence or presence of traveling patterns 
encodes values of Boolean logical variables. The 
trajectories of patterns approaching a collision site 
represent input variables, and the trajectories of 
the patterns ejected from a collision, and traveling 
away from the collision site, represent the results 
of logical operations, output variables.

A sub-excitable Belousov-Zhabotinsky 
medium (Sedina-Nadal et al, 2001) is an ideal 
substrate to build collision-based arithmetical 
chips in chemical systems. In a normal, excita 
table, mode the Belousov-Zhabotinsky medium 
responds to local perturbations by forming target 
or spiral waves, which propagate in all possible 
directions away from perturbation site. In a sub-excitable mode, we observe generation of 
a localized excitations, or wave-fragments that 
preserve their shape and travel like dissipative 
solitons (Bode et al, 2002) in one pre-determined 
direction for a substantial amount of time. We have 
demonstrated (Adamatzky, 2004; De Lacy 
Costello and Adamatzky, 2005; Adamatzky and 
De Lacy Costello, 2007; Toth et al, 2008) that it 
is possible to implement logical gates by collid 
ing excitation wave-fragments. In present article 
we use a cellular-automaton sub-excitable lattice 
(Adamatzky, 1995; Adamatzky, 1998) to simulate 
a sub-excitable chemical medium. We integrate 
together our previous results (Zhang and Ad 
amatzky, 2008; Zhang and Adamatzky, 2008) on 
arithmetical operations in collision-based media.

**CELLULAR-AUTOMATON MODEL 
OF SUB-EXCITABLE MEDIA**

We employ a two-dimensional three-state cellular 
automaton model of an excitable medium — the 
2+-medium, originally introduced in (Adamatzky, 
1995; Adamatzky, 1998). The 2+-medium consists 
of an orthogonal array of finite automata, where 
every automaton, called a cell, takes three states: 
resting, excited ‘+’ and refractory ‘-’ and updates 
its state depending on the states of its eight neigh 
bors. All cells update their states simultaneously 
and in discrete time, using the same rule. A resting 
cell becomes excited if it has exactly two excited 
neighbors. The transitions from excited state to 
refractory state, and from refractory state to rest 
ning state are unconditional.

Most common, and also minimal in size, local 
zations observed in 2+-medium, are particles 
traveling along horizontal and vertical directions 
of the cellular lattice (e.g. Figure 1a) and along 
diagonals of the lattice (e.g. Figure 1b). A 2+-par 
ticle consists of two excited cell-states and two 
refractory cell-states (Figure 1a). The particle 
can travel in four possible directions: East, West, 
North and South, and the particle conserves its 
configuration. A 3+-particle consists of three cells 
in excited states and three cells in refractory states 
(Figure 1b). The 3+-particle travels in the direc 
tion of North-East, South-East, North-West and 
South-West. Its topology changes in a cycle of 
four configurations.

There is the only one stationary localization 
in 2+-medium. The stationary oscillator (Figure