Online Methods for Portfolio Selection

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Abstract

This chapter overviews recent online portfolio selection strategies for financial markets. These investment strategies achieve asymptotically the same exponential rate of growth as the portfolio that turns out to be best ex post in the long run and do not require any underlying statistical assumptions on the nature of the stock market. The experimental results, which compare the performance of these strategies with respect to a standard sequence of historical data, demonstrate a high future potential of online portfolio selection algorithms.

Introduction

The purpose of this chapter is to overview some recent online portfolio selection sequential investment strategies for financial markets. A strategy must choose a portfolio of stocks to hold in each trading period, using information collected from the past history of the market. We consider the case when a portfolio selection strategy achieves asymptotically the same exponential rate of growth as the portfolio that turns out to be best ex post in the long run.
One of the most common approaches to adaptive investment strategies is probably the
distributional method proposed by Kelly in his work on horse race markets (Kelly, 1956).
In mathematical finance literature the resulting portfolio is often called the growth optimal
portfolio (Iyengar, 2005; Platen, in press). This approach assumes the existence of an
underlying probability distribution of the price relatives and uses Bayes’ decision theory
to specify the next portfolio. Under different conditions it was established that it is
possible to specify a sequence of investment strategies which is growth optimal for
ergodic and stationary markets with general assets return (Algoet & Cover, 1988).
However, in all these settings the optimal portfolio depends on the underlying distribu-
tion of the price relatives, which is usually unknown in practice.

In this chapter, we consider portfolio selection strategies that do not depend upon
underlying statistical assumptions (not even a probability distribution) and still achieve
asymptotically the same exponential rate of growth as the portfolio that turns out to be best ex post. Such an investment strategy was named a universal portfolio since the
convergence of the growth rate to the best ex-post rate is not done in a stochastic sense
but rather uniformly over all strategies that could possibly be optimal for a given market.
It was originally discovered in Cover (1991), Vovk (1990), and Helmbold, Schapire, Singer,
and Warmuth (1998), and later studied and applied in Cover and Ordentlich (1996),
Borodin and El-Yaniv (1998), Vovk and Watkins (1998), Cross and Barron (2003), and
others.

Financial institutions use a number of portfolio selection strategies, which, in general,
can be classified as passive investing and active investing. Passive investing is based
on the idea of market efficiency. A passive investor captures the market rate of return by
investing in a well diversified portfolio selected to match the performance of a standard
security market index. Passive investors rely on their belief that in the long term the
investment will be profitable. These strategies are usually called buy-and-hold (BAH).
The best ex post (also called optimal off-line) BAH strategy performs as well as the best
security in the market.

Active investing is an attempt to beat the overall returns of the security market by buying
and selling securities more frequently. This can be achieved through the purchase of
individual securities or actively-managed mutual funds. The active investing approach
assumes that there are inefficiencies in the market pricing of securities that can be
exploited by knowledgeable investors.

A typical active strategy that is considered in this chapter and is often used in practice
is the constant rebalanced portfolio (CRP). This strategy fixes a distribution of capital
over securities and uses it for every period. Therefore, it may be necessary to buy and
sell a security between periods in order to keep the proportion of one’s capital in each
security constant. For example, suppose we have two securities and the CRP (0.5, 0.5)
keeps the same amount of money in both of them in every trading period. Assume that the first
security is risk-free, that is, the price of it never changes, and the second security is highly
volatile. The price of the second security alternatively doubles and halves on even and
odd days respectively. When the second security doubles, the CRP (0.5, 0.5) relative return
grows by $\frac{1}{2} + \frac{1}{2} \times 2 = \frac{3}{2}$ since half of its money is in this security. When the second security