Reducing the 0-1 Knapsack Problem with a Single Continuous Variable to the Standard 0-1 Knapsack Problem

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ABSTRACT

The 0-1 knapsack problem with a single continuous variable (KPC) is a natural extension of the binary knapsack problem (KP), where the capacity is not any longer fixed but can be extended which is expressed by a continuous variable. This variable might be unbounded or restricted by a lower or upper bound, respectively. This paper concerns techniques in order to reduce several variants of KPC to KP which enables the authors to employ approaches for KP. The authors propose both, an equivalent reformulation and a heuristic one bringing along less computational effort. The authors show that the heuristic reformulation can be customized in order to provide solutions having an objective value arbitrarily close to the one of the original problem.

Keywords: Binary Knapsack Problem, Mixed Integer Programming, Reformulation, Single Continuous Variable (KPC), Variable Capacity

1. INTRODUCTION

The 0-1 knapsack problem with a single continuous variable (KPC) is a natural extension of the binary knapsack problem (KP), a well-known combinatorial optimization problem with applications in production, logistics, and distribution planning. The KP is to choose items in order to maximize profit without exceeding a given capacity. While the capacity cannot be influenced according to the KP the KPC considers the opportunity to extend or reduce, respectively, the available capacity. Extending capacity reduces the profit while reducing capacity increases the profit. The KP is well-known to be binary NP-hard, meaning that it can be solved in pseudo-polynomial time, see Garey and Johnson (1979) and Pisinger (2005). NP-hardness of KPC is straightforward. There is a huge amount of real life applications for the KP, e.g., the optimization of inventory policies, see Gorman and Ahire (2006). But the main importance is the use as a “building block”, occurring as a subproblem of a more complex problem. The same yields for the KPC as well.
In addition, a direct application for the KPC is the determination of the optimal choice of investment projects with a given budget, where the budget can be widened on credit. Although the problem setting is rather abstract solution methods for KPC yield decision support in this field in various ways. First, of course, for given parameters an optimal composition for an investment portfolio can be achieved. Second, we can accomplish a sensitivity analysis of optimality of specific portfolios with respect to the cost occurring when the budget is widened. Clearly, interest rates may vary over time. In order to measure the risk implied by variable interest rates we can solve our problem for different interest rates. Then, we can classify investment projects according to the range of interest rates under which they should be chosen. We obtain a scenario dependent investment strategy. Finally, provided we can evaluate likelihood for scenarios we can aim at finding portfolios having robustness measures in mind. The KP has been studied by numerous researchers during the last decades, see Kellerer et al. (2004) for example. Martello et al. (1999) develop an approach called combo, based on a combination of dynamic programming with tight bounds. Although published in 1999 it still seems to be the state-of-the-art, see Martello et al. (2000) and Pisinger (2005). Contrarily, very few papers concerning the KPC can be found. One proposal has been presented recently in Nauss (2004), which presents a branch&bound approach adopted from the KP. Another work is stated in Marchand and Wolsey (1997) along with Wolsey (2003) based on branch&cut: The linear programming relaxation of KPC is strengthened by adding additional constraints. The resulting upper bound is then used in a branch&bound procedure. However, a major drawback is the exponential number of cuts. The purpose of our work is to reduce the KPC to the KP which enables us to solve it employing the algorithm combo. The exposition of our work is as follows: In Section 2 we present the mixed integer formulation and develop several properties of solutions. The approaches for the one-sided and two-sided limitation of the capacity are given in Section 3 and Section 4, respectively. In Section 5 we provide some computational results and Section 6 concludes the paper.

2. MODEL FORMULATION AND FIRST INSIGHTS

Given a set \( N = \{1, \ldots, n\} \) of items with profit \( p_j \) and weight \( w_j \) for each item \( j \in N \), the objective is to maximize the profit sum of the chosen items regarding the capacity \( b, b > 0, \) of the knapsack. To this end, binary decision variable \( x_j, j \in N, \) is used.

\[
x_j = \begin{cases} 
1 & \text{if item } j \text{ is chosen} \\
0 & \text{otherwise}
\end{cases}
\]

We reasonably restrict the parameters: \( p_j > 0 \) and \( w_j > 0, j \in N. \) Moreover, in order to avoid trivial cases we assume \( \sum_{j \in N} w_j > b. \) According to the KPC the original capacity \( b, \) can be adjusted. Continuous variable \( s \) represents the amount of capacity which is added \( (s > 0) \) or removed \( (s < 0), \) respectively. The value per unit of flexible capacity is \( c, c > 0. \) Now, the problem can be stated mathematically as an extension of the classical formulation of KP, see, e.g., Pisinger (2005):

\[
\max \sum_{j \in N} p_j x_j - c \cdot s \quad (1)
\]

\[
s.t. \sum_{j \in N} w_j x_j \leq b + s \quad (2)
\]

\[
x_j \in \{0, 1\} \quad \forall j \in N \quad (3)
\]

\[
l \leq s \leq u \quad (4)
\]

The objective function (1) is twofold. The first part represents the sum of chosen items' profits. The second part considers cost and sales when additional capacity is bought and superfluous capacity is sold, respectively. Constraint (2)
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