Chapter 16
Analysis of State Dependent Vacation Queues with Threshold Gated Service Policy

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ABSTRACT
In this article, the authors introduce a simple approach for modeling and analyzing a $SII/G/1$ queue where the server may take repeated vacations. When a busy period ends, the server takes a vacation of random duration. At the end of each vacation, the server may either start a new vacation or resume service. If a queue is found of less than $n$ customers, the server will always take a new vacation. If there are at least $n$ customers in queue, the server provides services to those customers after a brief set-up time. The authors obtain several performance measures of the system, including the mean and second moment of the cycle time, the number of customers in a cycle of service, and the expected delay experienced by a customer.

INTRODUCTION
Queueing models with server repeated vacations have found applications in many areas such as computer science, telecommunication engineering, production and inventory networks. Server vacations are useful for the system in which the server wants to utilize his idle time for alternative purposes. For comprehensive and excellent surveys on queueing systems with server vacations, we refer the readers to Doshi (1986, 1990) and the books by Takagi (1991). It is usually assumed that the server becoming available, or unavailable, completely depends on the number of the customers in the system. For exhaustive service policy, the server is assumed to go on vacation when...
ever the system is empty. The instance at which the server comes back from a vacation and finds at least $N$ (predetermined threshold) customers in the system he begins serving immediately and exhaustively. This type of control policy is commonly known as $N$ policy queueing systems with vacations. We refer the readers to Kella (1989) and Lee and Srinivasan (1989) for detail discussions concerning $n$ policy $M/G/1$ and $M^x/G/1$ queueing systems with vacations, respectively. One can also find some applications of $N$ Policy queueing system in these two papers. For other types of nonexhaustive service policy we again refer the readers to Doshi (1986,1990) and the books by Takagi (1990, 1991).

The server setup or startup corresponds to the preparatory work of the server before starting the service. In some situations, the server often requires a startup time before starting each service period. Concerning queueing systems combining $N$ policy with startup time, Baker (1973) first proposed the $n$ policy $M/M/1$ queueing system with exponential startup time. Borthakur et al. (1987) extended Baker’s results to the general startup time. The $N$ policy $M/G/1$ queueing system with startup time was first studied by Minh (1988) and was investigated by several researchers such as Medhi and Templeton (1992), Takagi (1993), Lee and Park (1997), Ke (2001). Hur and Paik (1999) examined the operation characteristics of $M/G/1$ queueing system under $N$ policy with server startup and explained how the system’s optimal policy and cost behave for various arrival rates. It is to be noted that fewer researchers studied queueing systems under $N$ policy with server startup time and vacation time treated as different random variables. Furthermore, the exhaustive service policy is not adequate to capture observed phenomenon such as the server needs to be retool (or do some book keeping) after serving each batch of customers. Another example where $N$ policy fails can be observes from the tourism industry. A common observe phenomenon in the tourism industry is that guided tours are operating on the basis of if there are enough subscribers. Anyone interested in buying a guided tour usually books that service with the understanding that the tour maybe cancel if there are not enough subscribers. Once it is determine that there is enough subscribers, the dispatcher would send a smaller van or bigger bus to the predetermined meeting place depending on the number of subscribers at the schedule time. The travel time it takes the van or bus to travel to the predetermined meeting place may be considered as the server setup time. Customers who arrive during this period would be inform that this particular guided tour have been fill up and be routed to the next available guided tour. Clearly, this is an example where the exhaustive service nature of the $N$ policy doesn’t work. Therefore, there is a need to take into consideration non-exhaustive service policies. To the best of our knowledge, no papers have focus on state-dependent vacation models with nonexhaustive threshold trigger service policy. This could be partially explain by the fact that vacation queues with threshold trigger non-exhaustive service policy may not be analyze via straightforward application of the traditional method of analyzing vacation queues (e.g., transform techniques and busy period analysis). Motivated by the above observations, this paper aims to contribute to the theory of vacation models with threshold trigger nonexhaustive service disciplines and state dependent vacation for the server. Our model improves upon the existing literature by explicitly consider non-exhaustive service policy that takes into consideration the fact that server may have to engage in secondary job before they start serving customers again. The secondary job has to be completed first even if there is some customers in the systems at the end of servicing a batch of customers. We believe that our results are of interest to practitioners as well as academicians who are interested in managing queueing systems with unreliable or removable server.