Chapter 3
A New Technique for Estimating the Distribution of a Stochastic Project Makespan

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ABSTRACT
A critical success dimension in projects is the ability to complete a project within an estimated duration. In that regard, effective project scheduling techniques in an uncertain environment is of interest in many organizations. In this paper, the authors use an analytic approach to analyze the behavior of time duration distributions of projects in stochastic activity networks, and propose a simple computation scheme for approximating their distribution. The findings offer an understanding of the large gap between PERT and simulation results, and the deviation of projects from their intended schedules. In addition to providing theoretical framework, the proposed approach also recommends a simple practical pragmatic technique that computes the time distribution of project duration. This is a simple and handy tool for the project manager that may replace simulation. As a byproduct, the earliest start time distribution for each activity is also estimated.

INTRODUCTION
Scheduling is a core project management area, and finding project’s duration (makespan) is one of its important objectives (Kerzner, 2006; PMI, 2008; Zwikaæl et al., 2006). This paper presents a practical technique that estimates the makespan distribution of a project in a more accurate and reliable way than PERT (Project Evaluation and Review Technique), and is consistent with Monte Carlo Simulation results.
One of the most common techniques for project scheduling under uncertainty is PERT, which was originally proposed by Malcolm et al. (1959). While it is still a prevalent planning and controlling tool in project management (Ash & Pittman, 2008; Adler & Smith, 2009), it has been critiqued that PERT provides inaccurate information about the project completion time (e.g., Klingel, 1966; Shogan, 1977; Schonberger, 1981; Dodin, 1985; Schmidt & Grossman, 2000; Dodin, 2006; Hahn, 2008; Kirytopoulos et al., 2008). Alternatively, Monte Carlo simulation has been a practical tool for evaluating the makespan distribution of a stochastic project (e.g., Van Slyke, 1963; Burt & Garman, 1971; Sullivan et al., 1982; Iida, 2000; Demulemeester & Herroelen, 2002; Kirytopoulos et al., 2008). However, the simulation computational intensity and the time consumed by a simulation, motivated research for searching alternatives (Adlakha & Kulkarni, 1989; Cohen & Zwikael, 2008; Salaka & Prabhu, 2008). While several research directions had been pursued (as discussed in section 1.1.) there is still a need for a practical technique that would replace PERT and would give consistent results with Monte Carlo Simulation.

**Alternative Approaches and Their Shortcomings**

A common research direction has been to find the likelihood of an activity to become critical (Dodin & Elmaghhraby, 1985; Bowman, 1995; Cho & Yum, 1997; Elmaghhraby, 2000; Bowman, 2001). Another direction has been to find several possible critical paths (Dodin, 1984; Chen, 2007.) Iida (2000) identifies three other major research directions for evaluating a project’s makespan:

1. Analytical approaches to determine the distribution function of the project duration with diverging complexity (e.g., Hagsrom, 1990; Dodin & Sirvanci, 1990; Dodin, 2006.)
2. Analytical approaches for computing of the bounds on the distribution function of the project duration (e.g., Kleindirfer, 1971; Shogan, 1977; Dodin, 1985; Weiss, 1986; Ludwig et al., 2001.)
3. Analytical approaches for computing of the bounds on the expected project duration (e.g., Elmaghraby, 1967; Lindsey, 1972; Robillard & Trahan, 1976.)

So far, each of the above proposed approaches and techniques have its own shortcomings and consequently have been too complex or unattractive for broad adoption by project managers.

Starting from the first approach, the computational complexity of the method proposed by Hagstrom (1990) renders it impractical in most cases. Dodin (2006) proposed two distributions approximating two extreme cases: (1) For large parallel activity network the approximation to the distribution limit of infinite number of parallel critical paths is advised. This distribution is known as the Extreme Value (EV) distribution. (2) For more restricted network structure with only one critical path, the series approach of PERT is advised. However, most projects have structures very different than these two extremes. Moreover, Dodin does not explicitly addresses the uncertain risky events causing delays that current practices of risk management take into account while planning and setting the project completion time.

The second approach, of computing the bounds on the distribution, is not sufficient for scheduling decisions. Bounds tightness is hard to measure and tight bounds have different meaning depending on the shape of the distribution tails. Bounds do not allow seeing the whole picture, and to glean some important rules in estimating project duration.

The third approach, of computing of the bounds on the expected project duration is even more problematic. When a contract for executing a project is signed, usually the completion time is set so that a confidence level for completion is well beyond