Chapter 4

Constraint Handling in Particle Swarm Optimization

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ABSTRACT

In this article, the authors propose a particle swarm optimization (PSO) for constrained optimization. The proposed PSO adopts a multiobjective approach to constraint handling. Procedures to update the feasible and infeasible personal best are designed to encourage finding feasible regions and convergence toward the Pareto front. In addition, the infeasible nondominated solutions are stored in the global best archive to exploit the hidden information for guiding the particles toward feasible regions. Furthermore, the number of feasible personal best in the personal best memory and the scalar constraint violations of personal best and global best are used to adapt the acceleration constants in the PSO flight equations. The purpose is to find more feasible particles and search for better solutions during the process. The mutation procedure is applied to encourage global and fine-tune local searches. The simulation results indicate that the proposed constrained PSO is highly competitive, achieving promising performance.

INTRODUCTION

Recently, there are active studies on using particle swarm optimization (PSO) to solve constrained optimization problems (COPs). Similar to evolutionary algorithms (EAs), the original PSO design lacks a mechanism to handle constraints in an effective manner. Most of the constrained PSO designs adopted the popular constraint handling techniques that are built for EAs (Runarsson & Yao, 2005; Takahama & Sakai, 2006; Cai & Wang, 2006; Wang, Cai, Guo, & Zhou, 2007; Wang, Cai, Zhou, & Zeng, 2008). Evidence shows in recent publications on constraint handling with PSO including penalty methods (Parsopoulos & Vrahatis, 2002), comparison criteria or feasibility tourna-
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Constraint handling (Zielinski & Laur, 2006; He & Wang, 2007; Pulido & Coello Coello, 2004), lagrange-based method (Krohling & Coelho, 2006) lexicographic order (Liu, Wang, & Li, 2008), and multiobjective approach (Lu & Chen, 2006; Li, Li, & Yu, 2008; Liang & Suganthan, 2006; Cushman, 2007), to name a few. The reason of such popularity is credited to its simplicity, easy implementation and rapid convergence capability inherited in PSO design. Most constraint handling techniques such as penalty methods or comparison criteria are treated as an add-on module to be incorporated into any EAs for solving COPs. The PSO’s algorithm is built with mechanisms that can be exploited to handle constraints, without imposing any penalty methods or comparison criteria. Motivated by the advantages and its inherited ability, we propose a constrained PSO with design elements that exploit the key mechanisms to handle constraints as well as optimization of the objective function.

Consider a minimization problem; the general form of the COP is given as follows:

Minimize \( f(x) \), \( x = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^n \) \hspace{1cm} (1)

subject to

\[
g_j(x) < 0, \quad j = 1, 2, \ldots, m; \hspace{1cm} (2a)
\]

\[
h_j(x) = 0, \quad j = m + 1, \ldots, p; \hspace{1cm} (2b)
\]

\[
x_{i_{\text{min}}} \leq x_i \leq x_{i_{\text{max}}}, \quad i = 1, 2, \ldots, n; \hspace{1cm} (2c)
\]

where \( x \) is the decision vector of \( n \) decision variables. Its upper \( (x_{i_{\text{max}}}) \) and lower \( (x_{i_{\text{min}}}) \) bounds in Equation (2c) define the search space, \( S \subseteq \mathbb{R}^n \). \( g_j(x) \) represents the \( j \)th inequality constraint while \( h_j(x) \) represents the \( j \)th equality constraint. The inequality constraints that are equal to zero, i.e., \( g_j(x^*) = 0 \), at the global optimum \( (x^*) \) of a given problem are called active constraints. The feasible region \( (F \subseteq S) \) is defined by satisfying all constraints (Equations (2a)-(2b)). A solution in the feasible region \( (x \in F) \) is called a feasible solution, otherwise it is considered an infeasible solution.

In this article, we adopt a multiobjective constraint handling techniques in PSO for constraint handling since these techniques require neither penalty factors that need heuristic tuning nor balance the right proportion of selecting feasible and infeasible solutions in the population via selection criteria (Cai & Wang, 2006; Wang et al., 2007; Wang et al., 2008; Mezura-Montes & Coello Coello, 2006). Several design elements are incorporated into the proposed PSO (RCVPSO): 1) two updating procedures are designed to keep track of infeasible and feasible personal best separately; 2) infeasible nondominated solutions along with a best feasible nondominated solution are recorded in the global best archive; 3) the acceleration constants in the PSO flight equation is adjusted based on the feasibility ratio in the personal best memory and the constraint violations corresponding to the members in personal best memory and global best archive; and 4) a mutation operator applied in such the range of the upper and lower bounds of each decision variable is adaptively narrowed over time. This is to promote exploration in early search in order to find any feasible solutions, while encourage exploitation at the later search by local fine tuning in order to find the global optima.

The remaining sections complete the presentation of this article. In Section II, a review of relevant works in this area is presented. Section III elaborates on the proposed constrained PSO (RCVPSO). Comparative study and pertinent discussions are presented in Section IV. Finally, conclusion remarks are drawn in Section V.