Chapter 6

Fuzzy Judgments and Fuzzy Sets

Thomas L. Saaty
University of Pittsburgh, USA

Liem T. Tran
University of Tennessee, USA

ABSTRACT

Using fuzzy set theory has become attractive to many people. However, the many references cited here and in other works, little thought is given to why numbers should be made fuzzy before plunging into the necessary simulations to crank out numbers without giving reason or proof that it works to one’s advantage. In fact it does not often do that, certainly not in decision making. Regrettably, many published papers that use fuzzy set theory presumably to get better answers were not judged thoroughly by reviewers knowledgeable in both fuzzy theory and decision making. Buede and Maxwell (1995), who had done experiments on different ways of making decisions, found that fuzzy does the poorest job of obtaining the right decision as compared with other ways. “These experiments demonstrated that the MAVT (Multiattribute Value Theory) and AHP (Analytic Hierarchy Process) techniques, when provided with the same decision outcome data, very often identify the same alternatives as ‘best’. The other techniques are noticeably less consistent with the Fuzzy algorithm being the least consistent.”

1. INTRODUCTION

First linked to decision-making problems by Bellman and Zadeh (1970), the use of fuzzy sets in fuzzy multi-attribute decision-making (FMADM) methods is to deal with fuzzy data. Since the first classic FMADM method developed by Bass and Kwakernaak (1977), various FMADM methods have been developed (see Chen & Hwang, 1992; Triantaphyllou & Lin, 1996; Triantaphyllou, 2000; Figueira et al., 2004). In their testing and review, Triantaphyllou and Lin (1996) and Triantaphyllou (2000) found that each of the fuzzy decision-making methods under review yielded different
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rates of contradiction. In another review, Rao (2007) found that a majority of FMADM methods require cumbersome computations. Furthermore, they often force all elements, including those with crisp numbers in the decision matrix to be in a fuzzy format. This transformation not only goes against the intention of fuzzy set theory (e.g., no subjectivity introduced into precisely-known data) but also increases the computational burden and makes those FMADM methods hard to use.

Our purpose here is to show that whatever the claim of making numbers fuzzy may be, fuzzification does not necessarily improve the numerical value(s) of a solution in those situations when the true value is already known by other means and is being estimated by a numerical process (the principal eigenvector in the AHP) that represents judgments of involved participants, whether well or poorly informed. We give three different ways to show that fuzzy should not be used with the AHP and perhaps even more, it should not be used to make decisions because all judgments are not crisp measurements that are subject to fuzzy modification. We assume that the reader is familiar with the basic concepts of the AHP and thus summarize its main feature to use the principal eigenvector when the judgments are inconsistent. This is followed by a section that includes a few examples worked out particularly for this paper which show that the resulting fuzzy vector does not bring the values of the original eigenvector closer to the true answer. We then include a brief section that shows (proves) that judgments in the AHP are already fuzzy and hence insensitive to small changes in stimuli. Finally we show that a mathematical theory already exists which demonstrates that fuzzying AHP judgments is simply perturbing them and there is well-known theory which adequately explains how the outcome of perturbation of judgments is simply a small change in value without specifying that the result is closer or farther than the true value known in a particular problem and thus it can be made better or worse mindlessly. When shown to fuzzy set people they constantly ask for more fuzzifying without their own undertaking justification of our objections. Fuzzifying numbers wantonly is a hoax that resides in the lack of deeper understanding why human judgment is fuzzy. We believe strongly that fuzziness is the result of human limitations due to just noticeable difference (see below) and they have no hidden magic that lies in the simulations and manipulation of numbers in tedious ways. We urge the reader to look deeper into this observation.

2. FUZZY, AHP, EIGENVALUE AND EIGENVECTOR

Fuzzy set theory uses the AHP to drive fuzzy priorities that are already obtained by calculating the eigenvector. It relies on using the eigenvalue to improve inconsistence although it is known that a perfectly consistent matrix does not of necessity yield a valid result in that it is a best estimate of underlying measurements when such measurements are known. It is largely the quality of the judgments that determines the validity of the outcome and not their numerical precision. When the matrix is inconsistent we need the eigenvector to derive priorities.

In the field of decision-making, the concept of priority is quintessential and how priorities are derived influences the choices one makes. Priorities should be unique and not one of many possibilities; they must also capture the dominance of the order expressed in the judgments of the pairwise comparison matrix. The idea of a priority vector has much less validity for an arbitrary positive reciprocal matrix than for a consistent and a near consistent matrix. A matrix is near consistent if it is a small perturbation of a consistent matrix. The custom is to look for a vector \( w = (w_1, \ldots, w_n) \) such that the matrix \( W = (w_i/w_j) \) is “close” to \( A = (a_{ij}) \) by minimizing a metric. Metric closeness to the numerical values of the \( a_{ij} \) by itself says little about the numerical precision with which one element dominates another directly as in the