Chapter 8
Explanation Generation over Temporal Interval Algebra

Debasis Mitra  
Florida Institute of Technology, USA

Florent Launay  
Florida Institute of Technology, USA

ABSTRACT
Temporal interval algebra has generated strong interest for both theoretical and practical reasons. All its Maximal Tractable Subalgebras (MTS) have been identified. Now is the time to make the transition toward their practical applications. In this chapter, the authors have proposed a formalism on how to classify an input temporal network in one of these MTSs or decide its intractability. They have also proposed a linear algorithm for checking consistency when the input belongs to one of the seventeen MTSs, and for finding the constraints responsible for inconsistency in case the network is unsatisfiable.

INTRODUCTION
Interval Algebra (IA) is possibly the most studied algebra related to automated reasoning, for its theoretical elegance and feasible practical applications in scheduling, natural language engineering, etc. Some of the hallmark works in the area are: Vilain and Kautz’s (1986) proof of NP-hardness of IA; Nebel and Bürckert’s (1995) detecting the first maximal tractable algebra, namely, the ORD-Horn algebra (OH); Ligozat’s (1996) reinterpretation of that algebra in a canonical and geometrical representation space; and Krokhin et al.’s (2003) discovery of the exhaustive set of eighteen non-trivial Maximal Tractable Subalgebras (MTS). Sometimes we will use the word “algebra” or “subalgebra” synonymously with MTS.

A motivation behind studying the maximal tractable algebras is that an application domain may fall into one of these classes, or may be restricted to one of these classes making temporal reasoning more practical for that application. Given such an expectation it is only reasonable to ask, how can we identify an input system of interval constraints
whether it belongs to any of the MTSs? In this paper, we develop a classification structure of the MTSs and we propose an algorithm for identifying an input network if it belongs to any particular MTS. We use a novel geometrical interpretation of the MTSs for this purpose. Although, polynomial-time Path Consistency algorithm (PC) is complete for any MTS, we show that each MTS, other than the OH, has a similar behavior as the point-based reasoning problem (van Beek, 1992; Drakengren, et al., 1997), thus, enabling one to apply a more efficient cycle-checking algorithm than the PC. This new algorithm also has an extension for detecting culprit constraints when the input is inconsistent. Culprit detection is equivalent to the diagnosis as a task.

In the following section, we provide some background information on IA for uninitiated readers. Subsequently we will show the geometrical interpretation of the MTSs in Ligozat’s canonical space. We will then introduce the classification algorithm for an arbitrary interval constraint network. Lastly we provide the scheme for checking consistency of some Qualitative Temporal Constraint Networks (QTCNs) belonging to any of the MTSs, other than the OH.

**BACKGROUND ON TEMPORAL REASONING**

Qualitative reasoning with intervals involves thirteen atomic relations, \( B: \{ \text{before}(p), \text{after}(p') \), \text{meets}(m), \text{met-by}(m'), \text{overlaps}(o), \text{overlapped-by}(o'), \text{starts}(s), \text{started-by}(s'), \text{during}(d), \text{contains}(d'), \text{finishes}(f), \text{finished-by}(f') \), \text{equal(eq)} \}, \) between any pair of intervals (Allen, 1983). The corresponding relational algebra is comprised of the power set \( P(B) \), the power set of \( B \), which is closed under the traditional reasoning operators like composition, converse, set union, and set intersection.

**Definition 1: A Qualitative Temporal Constraint Network (QTCN)** is a graph \( G=(V, E) \), where each node denotes an interval, and each directed labeled edge \( (v_1, v_2, R) \in E \) represents disjunctive constraint \( R \) between \( v_1 \) and \( v_2 \), where \( R \in P(B) \). Two special relations are tautology (disjunction of all thirteen atomic relations or no constraint), and \( \emptyset \) (empty relation leading to inconsistency). Reasoning may be restricted to a subset \( \Theta \), where \( R \in \Theta \subseteq P(B) \), in case \( \Theta \) is closed under composition, converse and intersection, thus, forming a \( \Theta \)-subalgebra.

**Definition 2: Qualitative Temporal Reasoning Problem (QTR(\( \Theta \)))** is to answer, given a QTCN \( Q \) in which only relations form \( \Theta \) occur, if a satisfiable assignment for each of the nodes exists, such that all the constraints \( R \) in \( Q \) are satisfied. For \( \Theta = P(B) \), the full algebra is called the Interval Algebra or IA. The reasoning problem over full IA is known to be NP-hard (Vilain & Kautz, 1986). For \( \Theta \subset P(B) \), restricted reasoning is interesting if such a \( \Theta \)-subalgebra is tractable.

**Definition 3 (Maximal Tractable Subalgebra, or MTS):** A tractable subalgebra \( \Theta \) is maximal if it has no super-algebra \( \Theta' \) that is tractable, other than the full algebra \( P(B) \).

Eighteen MTSs for the IA have been found such that the list is exhaustive—no other MTS of IA exists (Theorem 2.3 of Krokhin, et al., 2003). This is true at least from linguistic perspective, i.e., we do not know if the graph structure of temporal network may provide more MTS. The list includes ORD-Horn MTS, which is the only MTS that includes all thirteen atomic relations in \( B \). We call the set of MTSs other than OH as the Krokhin-MTS.

Ligozat (1996) developed a canonical way of representing time interval relations geometrically, which appears as a useful tool for understanding the MTSs (Figure 1): An intervals is placed as a point in a 2D-Cartesian space, where the starting point is \( X \)-coordinate, the ending point is \( Y \)-coordinate, and the valid space is \( Y>X \), forbidding the interval ending point to occur before the starting point. For instance, if interval \( A \) ‘precedes’ interval \( B \), then \( A \) will be located as a point.
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