A Method for Ranking Non-Linear Qualitative Decision Preferences using Copulas

Biljana Mileva-Boshkoska, Jožef Stefan International Postgraduate School, Slovenia
Marko Bohanec, Jožef Stefan Institute and University of Nova Gorica, Slovenia

ABSTRACT

This paper addresses the problem of option ranking in qualitative evaluation models. Current approaches make the assumptions that when qualitative data are suitably mapped into discrete quantitative ones, they form monotone or closely linear tabular value functions. Although the power of using monotone and linear functions to model decision maker’s preferences is impressive, there are many cases when they fail to successfully model non-linear decision preferences. Therefore, the authors propose a new method for ranking discrete non-linear decision maker preferences based on copula functions. Copulas are functions that capture the non-linear dependences among random variables. Hence each attribute is considered as a random variable. The variables are nested into hierarchical copula structures to determine the non-linear dependences among all attributes at hand. The obtained copula structure is used for obtaining regression function and consequently for option ranking. The application of the method is presented on two examples.

Keywords: Copula, Multi-Attribute Decision Making, Option Ranking, Qualitative Decision Models, Qualitative-Quantitative Method

INTRODUCTION

Solving decision problems described with multiple attributes is part of our daily life. The main difficulties occur when the decision problems become too complex. The complexity of these problems increases in cases when:

- The number of attributes increase,
- The set of possible values that the attributes may receive increases (in general people may consistently distinguish five levels, such as five grades at school, while experts may consistently distinguish up to seven levels (Saaty, 2003)),
- Several options belong to the same qualitative class of preferable options, therefore it is difficult to identify the best one (when one has to choose from many varieties, the decision making becomes a difficult task (“The tyranny of choice,” 2010)), and
- The problem itself is ill-structured and it is impossible to examine it through single attribute, criteria or point of view that would

DOI: 10.4018/jdsst.2012040103
lead to the optimum decision (Zopounidis & Doumpos, 2006).

Addressing these issues in different decision problems evolved into development of different quantitative and qualitative multi-attribute decision analysis (MADM) methods and tools (Adam & Humphreys, 2008; Figueira et al., 2005; Bouyssou et al., 2006). Quantitative methods represent attributes with quantitative (numerical) values. The group of classical quantitative methods is large and includes, among others, outranking methods (ELECTRE and its variants, PROMETEE) (Figueira et al., 2005), methods based on Multiple Attribute Utility Theory (MAUT) (Jacquet-Lagreze & Siskos, 1982) and Analytical Hierarchy Process (AHP) (Saaty, 2008). Qualitative methods, on the other hand, represent attributes with qualitative (symbolic) values. Methods that belong to this group are ZAPROS (Moshkovich & Larichev, 1995), which is based on verbal decision theory, Rough Sets (Greco et al., 2001) and Doctus (Baracskai & Dörfler, 2003). This paper builds on the DEX method (Bohanec & Rajkovič, 1990), which is a member of the latter group. DEX has been successfully used in a wide range of applications, such as environmental (Bohanec et al., 2008), agricultural (Pavlovič et al., 2011), and in medicine and healthcare (Bohanec, Zupan, & Rajkovič, 2000). DEX is implemented in the computer program called DEXi (Bohanec, 2011).

In DEX, the aggregation of discrete qualitative attributes is specified with a table whose rows are interpreted as if-then rules. Specifically, the decision maker’s preferences over the available options are defined using an attribute that are called a qualitative class. Options that are almost equally preferred belong to the same qualitative class. Consequently, a partial ordering of options is obtained.

Qualitative evaluation of options suffers from two problems: it provides only partial ranking of options instead of full ranking, and is insensitive to small differences among options. One possible way to overcome these problems is to combine qualitative and quantitative evaluation. In addition to qualitative evaluation, which ranks options into classes, we wish to numerically rank options within classes. In this paper, we propose an approach that constructs such a quantitative evaluation model automatically from decision maker’s qualitatively specified preferences.

Our work starts with the examination of the Qualitative-Quantitative (QQ) method (Bohanec, Urh, & Rajkovič, 1992; Bohanec, 2006). QQ is an extension of DEX that was developed in order to address the stated problem. QQ maps qualitative attributes consistently into quantitative ones, which are evaluated yielding a numerical utility. QQ aggregates qualitative attributes that may be connected on one level or may build a hierarchical structure. Such a quantitative representation of the model brings the main asset of the method: it is used to distinguish among the options that belong to the same class by ordering them.

The applicability of QQ is limited to decision problems in which the qualitative attributes can be regarded as discrete variables forming monotone or closely linear tabular functions. In the tabular functions, each row represents an option that belongs to a class. To check the monotonicity, one has to examine the comparable options in the tabular function. Monotonicity implies that for each comparable pair of options, the one with higher attribute values will receive higher or at least equal class value as the other option. In QQ, options that belong in the same class are modelled with a linear function $f_c$ whose output values are in the range $c \pm 0.5$. A tabular function is considered closely linear if it can be ‘sufficiently well’ (by some distance measure) approximated by some linear function $f_c$.

In order to overcome the problem of evaluation of non-monotone decision preferences, we propose a new QQ-based method, which uses techniques that capture the non-linear dependencies among different attributes expressed qualitatively by the decision maker. Unlike conventional methods like correlation that summarize the linear dependence relation, we propose to use copulas (Nelsen, 2006). Copu-
Related Content

Knowledge Management and Sharing
www.igi-global.com/chapter/knowledge-management-sharing/8079?camid=4v1a

Prioritizing Lean Six Sigma Efforts Using Bayesian Networks
Yanzhen Li, Rapinder S. Sawhney and Joseph H. Wilck IV (2014). Analytical Approaches to Strategic Decision-Making: Interdisciplinary Considerations (pp. 77-91).
www.igi-global.com/chapter/prioritizing-lean-six-sigma-efforts-using-bayesian-networks/102151?camid=4v1a

Exploring the Determinants of Success among Ladies Golfers by DEA-SBM Model
Wan-Chun Hsiung and Pi-Heng Chung (2014). International Journal of Strategic Decision Sciences (pp. 87-98).
www.igi-global.com/article/exploring-the-determinants-of-success-among-ladies-golfers-by-dea-sbm-model/116463?camid=4v1a
A Knowledge Worker Desktop Model (KWDM) Applied to Decision Support System
Encyclopedia of Decision Making and Decision Support Technologies (pp. 584-592).
www.igi-global.com/chapter/knowledge-worker-desktop-model-kwdm/11298?camid=4v1a