Chapter 12
Pseudo-Cut Strategies for Global Optimization

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ABSTRACT
Motivated by the successful use of a pseudo-cut strategy within the setting of constrained nonlinear and nonconvex optimization in Lasdon et al. (2010), we propose a framework for general pseudo-cut strategies in global optimization that provides a broader and more comprehensive range of methods. The fundamental idea is to introduce linear cutting planes that provide temporary, possibly invalid, restrictions on the space of feasible solutions, as proposed in the setting of the tabu search metaheuristic in Glover (1989), in order to guide a solution process toward a global optimum, where the cutting planes can be discarded and replaced by others as the process continues. These strategies can be used separately or in combination, and can also be used to supplement other approaches to nonlinear global optimization. Our strategies also provide mechanisms for generating trial solutions that can be used with or without the temporary enforcement of the pseudo-cuts.

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INTRODUCTION

We consider the constrained global optimization problem \((P)\) expressed in the following general form:

\[
(P) \text{ minimize } f(x) \\
\text{subject to: } \\
G(x) \leq b \\
x \in S \subseteq \mathbb{R}^n
\]

where \(x\) is an \(n\)-dimensional vector of decision variables, \(G\) is an \(m\)-dimensional vector of constraint functions, and without losing generality the vector \(b\) contains upper bounds for these functions. The set \(S\) is defined by simple bounds on \(x\), and we assume that it is closed and bounded, i.e., that each component of \(x\) has a finite upper and lower bound.

We introduce strategies for solving \((P)\) which are based on pseudo-cuts, consisting of linear inequalities that are generated for the purpose of strategically excluding certain points from being admissible as solutions to an optimization problem. The \textit{pseudo} prefix refers to the fact that these inequalities may not be valid in the sense of guaranteeing that at least one globally optimal solution will be retained in the admissible set. Nevertheless, a metaheuristic procedure that incorporates occasional invalid inequalities with a provision for replacing them can yield an aggressive solution approach that can prove valuable in certain settings. In the case of zero-one variables, for example, a concave function such as \(x_j(1 - x_j)\) may be used that is 0 when \(x_j = 0\) or 1, and is positive otherwise. See Bowman and Glover (1972) for additional examples.

We make recourse to an independent algorithm to generate trial solutions to be evaluated as candidates for a global optimum, where as customary the best feasible candidate is retained as the overall “winner”. The independent algorithm can consist of a directional search (based on gradients or related evaluations) as in Lasdon et al. (2010), or may be a “black box” algorithm as used in simulation optimization as in April et al. (2006) and Better et al. (2007).

PSEUDO-CUT FORM AND REPRESENTATION

Our pseudo-cut strategy is based on generating hyperplanes that are orthogonal to selected rays (half-lines) originating at a point \(x'\) and passing through a second point \(x''\), so that the hyperplane intersects the ray at a point \(x_0\) determined by requiring that it lies on the ray at a selected distance \(d\) from \(x'\). The half-space that forms the pseudo-cut is then produced by the associated inequality that excludes \(x'\) from the admissible half-space. We is particularly motivated by the work of Lasdon et al. (2010), where a simplified instance of such strategies was found to be effective for improving the solution of certain constrained non-convex nonlinear continuous problems.

In the present paper we likewise assume the objective function of \((P)\) is non-convex (hence a local optimum may not be a global optimum), and allow for non-convexity in the constraints. We also allow for the presence of integer restrictions on some of the problem variables under the provision that such variables are treated by means of constraints or objective function terms that permit them to be treated as if continuous within the nonlinear setting. In the case of zero-one variables, for example, a concave function such as \(x_j(1 - x_j)\) may be used that is 0 when \(x_j = 0\) or 1, and is positive otherwise. See Bowman and Glover (1972) for additional examples.