Chapter 5
Routing

Kevin M. Curtin
George Mason University, USA

ABSTRACT
Routing is the act of selecting a course of travel. Routing problems are one of the most prominent and persistent problems in geoinformatics. This large research area has a strong theoretical foundation with ties to operations research and management science. There are a wide variety of routing models to fit many different application areas, including shortest path problems, vehicle routing problems, and the traveling salesman problem, among many others. There are also a range of optimal and heuristic solution procedures for solving instances of those problems. Research is ongoing to expand the types of routing problems that can be solved, and the environments within which they can be applied.

INTRODUCTION
Routing is the act of selecting a course of travel. This process is undertaken by nearly every active person every day. The route from home to school or work is chosen by commuters. The selection of stops one will make for shopping and other commercial activities and the paths between those stops is a routing activity. Package delivery services plan routes for their trucks in such a way that packages are delivered within specified time windows. School buses are assigned routes that will pick up and deliver children in an efficient manner. Less tangible objects such as telephone calls or data packets are routed across information networks. Routing is the most fundamental logistical operation for virtually all transportation and communications applications.

As in the examples above, routing is frequently seen as a practical effort to accomplish some goal. Its importance to geoinformatics, however, lies in the nature of routing as a general problem. Transportation, communications, or utility systems can all be modeled as networks—connected sets of edges and vertices—and the properties of networks can be examined in the context of the mathematical discipline of graph theory. Routing procedures can be performed on any network dataset, regardless of the intended application. This chapter will discuss the formulation of routing problems including shortest path problems, and will review in detail general vehicle routing problems and the travel-
Routing salesman problem. Solution procedures for routing problems are discussed and future trends in routing research are outlined.

**BACKGROUND**

Generally, a routing procedure is based on an objective—or goal—for the route, and a set of constraints regarding the route’s properties. By far the most common objective for routing problems is to minimize cost. Cost can be measured in many different ways, but is frequently defined as some function of distance, time, or difficulty in traversing the network. Thus the problem of locating the least cost or shortest path between two points across a network is the most common routing problem. It is also a problem for which there are several extremely efficient algorithms that can determine the optimal solution. The most widely cited algorithm that solves the least cost path problem on directed graphs with non-negative weights was developed by Edsger Dijkstra (1959), and an even more efficient version of this algorithm—the two-tree algorithm—exists (Dantzig, 1960). Alternative algorithms have been presented that will solve this problem where negative weights may exist (Bellman, 1958), where all the shortest paths from each node to every other node are determined (Dantzig, 1966; Floyd, 1962), and where not only the shortest path but also the 2nd, 3rd, 4th, or kth shortest path must be found (Evans & Minieka, 1992).

**NETWORK DESIGN PROBLEMS**

The shortest path problem is just one of a class of related routing problems that can be described as network design problems. Network design problems require that some combination of the elements of a network (edges and vertices) be chosen in order to provide a route (or routes) through the network. This group includes the minimal spanning tree problem, the Steiner tree problem, the Traveling Salesman Problem, and the vehicle routing problem, among many others (Magnanti & Wong, 1984). The modeling of these problems frequently takes the form of integer programming models. Such models define an objective and a set of constraints. Solution procedures are applied that require decisions to be made that generate a route that optimizes the objective while respecting the constraints. Given the limited space in this forum, the following sections will focus on the modeling of two significant routing problems in an effort to demonstrate the characteristics of the general class. Vehicle Routing Problems are presented in order to discuss the range of possible objectives for routing problems, and the Traveling Salesman Problem is presented to demonstrate the formulation of the objectives and constraints.

**Vehicle Routing Problems**

Vehicle Routing Problems (VRPs) are those that seek to find a route or routes across a network for the delivery of goods or for the provision of transport services. From their earliest incarnations VRPs have been formulated as distance or cost minimization problems (Clarke & Wright, 1964; Dantzig & Ramser, 1959). This overwhelming bias persists to this day. Nine out of ten research articles regarding route design in the context of transit routing written between 1967 and 1998 and reviewed by Chien and Yang (2000) employed a total cost minimization objective. When the route is intended as a physical transport route, the cost objective is nearly always formulated as a generalized measure of operator costs (List, 1990), user costs (Dubois et al., 1979; Silman et al., 1974), or both operator and user costs (Ceder, 2001; Chien et al., 2001; Lampkin & Saalmans, 1967; Newell, 1979; Wang & Po, 2001).

The few exceptions include a model that maximizes consumer surplus (Hasselström, 1981), a model that seeks to maximize the number of public transport passengers (van Nes et al., 1988), a model