Chapter 5
Robot Motion Models

ABSTRACT
This is the first chapter of the second section, a section devoted to mobile robot localization. Before presenting the general Bayesian framework for that problem at chapter 7, it is first required to study the different probabilistic models of robot motion. This chapter explores some of the reasons why any real robot cannot move as perfectly as planned, thus demanding a probabilistic model of the robot actions—mainly, its movements. Special emphasis is put on the most common ground wheeled robots, although other configurations (including non-robotic ones) with more degrees of freedom, such as arbitrarily-moving hand-held sensors or aerial vehicles, are also mentioned. The best-known approximate probabilistic models for robot motion are provided and justified.

CHAPTER GUIDELINE
• You will learn:
  ◦ The particular place of odometry and proprioceptive sensors in the Bayesian filtering framework.
  ◦ Six different kinematic models and their corresponding motion models with uncertainty, including closed-form formulas for the case of all random variables being Gaussians.
• Provided tools:
  ◦ A table which relates kinematic models with physical locomotion implementations.
  ◦ A systematic method to obtain a probabilistic counterpart of any ideal kinematic model under the assumptions of Gaussian distributions.
  ◦ Practical formulas to settle hard-to-tune parameters in the constant velocity and the no-motion models.
• Relation to other chapters:
  ◦ Kinematic model equations are directly applicable to the particle filter methods introduced in chapters 7 and 9 for localization and map building, respectively.
  ◦ Probabilistic motion models for Gaussian models find their utility for localization and mapping when using parametric distributions in chapters 7, 9, and 10.

DOI: 10.4018/978-1-4666-2104-6.ch005
1. INTRODUCTION

In the previous chapters, we have studied the mathematical foundations needed for addressing the problems of mobile localization and mapping. Now it is time to turn again to our robotic realm and begin to set the particular bases for solving robotics problems.

As explained in chapter 1, as a mobile robot moves throughout its environment, it constantly needs to update estimation about its instantaneous pose (i.e. the problem of localization) and, if the environment itself is unknown, a model of the world around (i.e. the problem of mapping; tackling both problems together is SLAM). In both cases the robot must rely on the readings from its sensors, both exteroceptive and proprioceptive, to perform these updates.

Both localization and SLAM consist of dynamic processes, thus we can apply recursive Bayesian estimation, introduced in chapter 4 section 9, when dealing with them. Recall that recursive here means iteratively computing the belief about some unknowns $x_k$ from the belief for the previous time step, that is, $x_{k-1}$. Most probabilistic models of robot motion can be directly applied to either localization or SLAM; the only difference will be in the vector of unknowns $x_k$, which in localization comprises the pose of the robot while in SLAM additionally includes a parameterized model of the world. In any case, we showed in chapter 4’s Equation 29 that the update equation for this recursive Bayesian estimator can be written as:

$$
\begin{align*}
\text{posterior at step } k & \propto \text{likelihood} \\
& \propto \int_{x_{k-1} = -\infty}^{+\infty} f_{x|x_{k-1}}(x_k | u_{1:k}) \, dx_{k-1} \\
& \propto \int_{x_{k-1} = -\infty}^{+\infty} f_{z | x_{k-1}}(z_k | x_{k-1}) \, dx_{k-1}
\end{align*}
$$

Equation 1 stands for a generic Bayesian estimation problem where the only information is the sequence of observations $z_{1:k}$. As it will be detailed in chapters 7 and 9, in localization and SLAM we will also have knowledge about another sequence of variables: the robot actions, denoted as $u_{1:k}$—we already briefly discussed robot actions in chapter 2. It will be shown there that recursive Bayesian estimation in those problems leads to transition models conditioned not only on the previous pose, but on the latest action as well. That is, we have to define $f_{x_k|z_{1:k},u_{1:k}}$ instead of $f_{x_k|z_{1:k}}$. As we will see next, the actual meaning of these action vectors $u_{1:k}$ depends on the kinematics of the robot and the particular instrumentation of its motion. Typically, if the robot is equipped with some sort of odometry, it simply consists of its readings.

Notice that odometry is provided by sensors, which implies that the transition must include sensory information, in addition to the sensory information included in the observation likelihood part of Equation 1. At first sight, there exist no evident reasons for treating each kind of sensor differently in the recursive update equation. However, under a Bayesian approach such as the one at hand some differences start to emerge. We will distinguish two types of data inputs to the formulation: on the one hand, the transition model will be devised from the kinematics of the robot being particularized with data coming