Chapter 3

Recurrence Quantification Analysis of Financial Markets

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ABSTRACT

Recurrence quantification analysis is a nonlinear time series analysis technique that detects deterministic dependencies in time series. This technique is particularly appropriate for modeling financial time series since it requires no assumptions on stationarity, statistical distribution, and minimum number of observations. This chapter illustrates two applications of recurrence quantification analysis to financial data: a set of international stock indices, and zero-coupon yields of US government bonds.

INTRODUCTION

A good understanding of asset price dynamics is crucially important for portfolio and risk management purposes, as well as for derivatives pricing. While in classical financial economics, the dynamics of asset prices is typically described by homoscedastic random walks or heteroscedastic martingale difference sequences, simple nonlinear deterministic processes can emulate price dynamics that are indiscernible from stochastic processes, providing an alternative model for the behavior of asset prices. Furthermore, nonlinear deterministic processes may explain the large fluctuations observed in financial data that stochastic models cannot account for (Hsieh, 1991). While there is compelling statistical evidence that asset prices do not always follow random walks (e.g., Lo & MacKinlay, 1988), there is little agreement whether the dynamics of financial data is consistent with stochastic or deterministic processes (Barnett et al, 2000).

Recurrence plots (Eckmann et al., 1987) and recurrence quantification analysis (Zbilut & Webber, 1992; Webber & Zbilut, 1994; Marwan et al, 2002) are nonlinear time series analysis techniques that detect deterministic dependencies in time series data. A recurrence plot (RP) is a visual representation of recurrences (similar system states attained at distinct times) that reveals deterministic behavior in the motion of a dynamical system. Recurrence quantification analysis (RQA) provides...
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the instruments for quantification of recurrences and detect critical transitions in the system. These techniques are particularly appropriate for testing for determinism in financial data since they require no assumptions on stationarity, statistical distribution and minimum number of observations. In recent years, several studies employed RPs and RQA to detect deterministic dependencies in financial markets. These studies contemplated markets such as stocks (Holyst et al, 2001; Zbilut, 2005; Fabretti & Ausloos, 2005; Guhathakurta et al, 2010, Bastos & Caiado, 2011), exchange rates (Belaire-Franch et al, 2002; Strozzi et al, 2002; Strozzi et al, 2007) and electricity prices (Strozzi et al, 2008).

This chapter presents a short introduction to recurrence plots and recurrence quantification analysis, and illustrates two empirical applications of this methodology to financial data. First, the dynamics of a large set of international equity indices is examined. It is shown that RQA reveals fundamental differences in the dynamics of stocks across different markets, particularly when developed markets are compared to their emerging counterparts. Then, the term structure of US government bond yields is analyzed. This analysis shows that zero-coupon yields across different maturities have distinct dynamics.

The next section provides a description of recurrence plots. The following section introduces some of the measures employed in the their quantification. This is followed by an application of this methodology to the aforementioned financial data. The last section provides some concluding remarks.

RECURRENCE PLOTS

This section provides a brief introduction to recurrence plots. A comprehensive description can be found in Marwan et al. (2007). As mention in the introductory section, recurrence plots depict the different occasions when dynamical systems visit the same region of phase space. Given a scalar time series \{x(i): i = 1,...,N\}, a recurrence plot is constructed by first “embedding” the series into a multi-dimensional space of vectors whose coordinates are the present and lead values of the series, \(v(i) = \{x(i), x(i+\tau), x(i+2\tau),..., x(i+(m-1)\tau)\}\).

The parameter \(m\) is called the embedding dimension and \(\tau\) is the time delay. According to Takens’ theorem (Takens, 1981), it is possible to reconstruct the original phase-space topology of a dynamical system from embedding vectors of univariate measurements of the system state, provided that the embedding dimension is sufficiently greater than the dimension of the underlying system. Second, a recurrence matrix of embedding vectors is constructed,

\[R_y(\varepsilon) = 0 \text{ if } ||v(i) - v(j)|| > \varepsilon,\]
\[R_y(\varepsilon) = 1 \text{ if } ||v(i) - v(j)|| \leq \varepsilon,\]

with \(i,j = 1,\ldots,N-(m-1)\tau, || \cdot ||\) is a norm, typically taken as the \(L_2\)-norm (Euclidean norm), and \(\varepsilon\) is some threshold distance.

A recurrence plot is obtained by placing a dot at coordinate \((i,j)\) of a two-dimensional plane when \(R_y(\varepsilon) = 1\), that is, when vector \(v(i)\) is close to \(v(j)\). Since \(R_y(\varepsilon) = 1\), the main diagonal line of the RP, called the line of identity, consists entirely of recurrence points. Furthermore, the plots are symmetrical with respect to the line of identity since \(R_y(\varepsilon) = R_y(\varepsilon)\). Patterns formed by adjacent recurrence points provide evidence for determinism and periodicity in the system. Diagonal lines parallel to the line of identity occur when segments of the trajectory visit the same region of the phase space at distinct times. The length of these lines is determined by the duration of these visits. Vertical or horizontal lines suggest stationary states in which the system persists in the same region for some time. On the other hand, isolated recurrence points may occur when states are rare, show little persistency or large fluctuations. While deterministic systems tend