Chapter 16
Petri Nets and Discrete Events Systems

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ABSTRACT
This paper provides an overview over the relationship between Petri Nets and Discrete Event Systems as they have been proved as key factors in the cognitive processes of perception and memorization. In this sense, different aspects of encoding Petri Nets as Discrete Dynamical Systems that try to advance not only in the problem of reachability but also in the one of describing the periodicity of markings and their similarity, are revised. It is also provided a metric for the case of Non-bounded Petri Nets.

INTRODUCTION
The interaction between perception and/or memorization with information exposing time are consistent with a theory which relates processing time to perceived duration, see Thomas (1975). It is mainly due because such temporal information is obtained from both a sort of timer processing and a visual information processing, so attention effectiveness is a function of these two processings, in which the range of durations become so important that it has been proved that there is a minimum quantum of time (discrete measurements) below which neither work. These facts also open the field of considering timed formalisms (Pelayo, 2005) for properly describing and analyzing these issues.

In Wang (2002), a seed study of the Cognitive Models of the brain can be found. Among others, a description of the memorizing process is there shown, moreover Wang has given an implementation of this cognitive process in RTPA, which has been taken by the authors of this paper as reference in Barquilla (2008) to formally describe this cognitive process.
Petri Nets (Petri, 1966) and Dynamical Systems (Arrowsmith, 1990) have increased their popularity during the last decades. Both mathematical tools have many analysis methods available and have become classical for modeling discrete-event processes.

In fact, many papers (David, 1991, 1994; Silva, 1990) show how discrete-event processes can be modeled by Petri Nets. There are also papers that present how to model these processes by discrete-event dynamical systems (Foursov, 2006).

Thus, it motivates to analyze if there exists any relation between these tools and what advantages are taking each one. Mainly, we wonder if a given Petri net corresponds directly to dynamical systems and the Petri Net properties have counterparts in the corresponding dynamical system.

In Foursov (2006), the authors make a partial attempt to do that, i.e., based on formal power series, they obtain an algorithm that allows checking whether a given weighted Petri net corresponds to a continuous polynomial dynamical system. But in this context, different initial states of the Petri net (different initial markings) could correspond to different dynamical systems.

Therefore, the work of giving a correspondence between Petri nets and dynamical systems was incomplete and we proposed (Guirao, 2011) different ways that try to solve not only the problem of reachability, but also the one of describing the periodicity of markings and their similarity.

The structure of the paper is as follows: We show how Petri nets can be used to model discrete-event processes. It is explained how to define the state space and the evolution operator in order to encode Petri nets as discrete dynamical systems. The next section is dedicated to introduce a metric for the states space; it is based on the Bayre metric, usually employed when modeling computer processes. We deal with giving a proper metric for non-bounded Petri nets. Finally, some conclusions and further research directions are presented.

**PETRI NETS MODELING DISCRETE-EVENTS PROCESSES**

A Petri Net is a bipartite graph constituted by two kinds of nodes, namely, places and transitions that alternate on a path made up of consecutive arcs. Places are usually represented by circles and transitions by boxes or rectangles. The number of places is finite and not zero and the same occurs for the number of transitions (David, 1994). More rigorously:

**Definition 1:** A Petri Net (PN) is a triple $N = (P, T, F)$ consisting of two finite sets $P$ and $T$, and a relation $F$ defined over $P \cup T$, such that:

1. $P \cap T = \emptyset$
2. $F \subseteq (P \times T) \cup (T \times P)$
3. $\text{dom} (F) \cup \text{cod} (F) = P \cup T$

$P$ is said to be the set of places, $T$ is called the set of transitions and $F$ is named the flow relation.

A Petri Net is a very useful tool in order to model a concurrent system due to not only their graphical nature, but also because of being able to simulate concurrent execution of actions in a system, in this sense, Petri Nets are marked. That is, the state of a system described by a PN is captured by means of the so called Markings.

**Definition 2:** Let $N = (P, T, F)$ be a Petri Net. A function $M : P \rightarrow N$ is a Marking of $N$. Thus, $(P, T, F, M)$ is a Marked Petri Net, MPN.

Markings of Petri Nets are graphically represented by including in the places as many points as tokens.

Given a MPN $(P, T, F, M)$ with $P = \{p_1, \ldots, p_n\}$, a marking $M$ of it is codified as the n-tuple containing in position $i$ the number...
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