Chapter 6
Data Envelopment Analysis with Fuzzy Parameters: 
An Interactive Approach

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ABSTRACT

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. In the conventional DEA, all the data assume the form of specific numerical values. However, the observed values of the input and output data in real-life problems are sometimes imprecise or vague. Previous methods have not considered the preferences of the decision makers (DMs) in the evaluation process. This paper proposes an interactive evaluation process for measuring the relative efficiencies of a set of DMUs in fuzzy DEA with consideration of the DMs’ preferences. The authors construct a linear programming (LP) model with fuzzy parameters and calculate the fuzzy efficiency of the DMUs for different α levels. Then, the DM identifies his or her most preferred fuzzy goal for each DMU under consideration. A modified Yager index is used to develop a ranking order of the DMUs. This study allows the DMs to use their preferences or value judgments when evaluating the performance of the DMUs.

INTRODUCTION

The changing economic conditions have challenged many organizations to search for more efficient and effective ways to manage their business operations. Data envelopment analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous decision making units (DMUs) by evaluating their relative efficiency. A DMU is considered efficient when no other DMU can produce more outputs using an equal or lesser amount of
inputs. The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs. The traditional DEA methods such as CCR (Charnes et al., 1978) and BBC (Banker et al., 1984) require accurate measurement of both the inputs and outputs. However, the real evaluation of the DMUs often exhibit imprecision and great uncertainty. In general, as the system’s complexity increases, exact evaluation of data becomes extremely difficult. In addition, the traditional DEA models generally do not consider the decision maker’s (DM’s) preferences or value judgments. Although a few researchers have considered the DM’s preferences, their model requires precise and exact measurement of both the input and output data (Joro et al., 2003; Wong et al., 2009; Yang et al., 2009).

In this study, we propose an interactive evaluation process for measuring the relative efficiencies of a set of DMUs in fuzzy DEA with consideration of the DMs’ preferences. We construct a linear programming (LP) model with fuzzy parameters and calculate the fuzzy efficiency of the DMUs for different α levels. Then, the DM identifies his/her most preferred fuzzy goal for each DMU under consideration. A modified Yager index is introduced and used to develop a ranking order of the DMUs. The main thrust of this study is to allow the DMs to use their preferences or value judgments when evaluating the performance of the DMUs. This paper is organized into seven sections. The next section presents a brief review of the existing literature on fuzzy DEA followed by a discussion of fuzzy number rankings. Then, we present an overview of fuzzy DEA. Following this overview, we illustrate the details of the proposed framework followed by a numerical example in order to demonstrate the applicability of the proposed framework and also to exhibit the efficacy of the procedures and algorithms. Finally, we conclude with our conclusions and future research directions.

**FUZZY DEA LITERATURE REVIEW**

In the conventional DEA, all the data assume the form of specific numerical values. However, the observed values of the input and output data in real-life problems are sometimes imprecise or vague. Imprecise evaluations may be the result of unquantifiable, incomplete and non-quantifiable information. The “Stochastic approach” and the “fuzzy approach” are two existence approaches for modeling uncertainty in the DEA literature. The stochastic approach involves specifying a probability distribution function (e.g., normal) for the error process (Sengupta, 1992). However, as pointed out by Sengupta (1992), the stochastic approach has two drawbacks associated with modeling the uncertainty in DEA problems:

1. Small sample sizes in DEA make it difficult to use stochastic models, and
2. In stochastic approaches, the DM is required to assume a specific error distribution (e.g., normal or exponential) to derive specific results. However, this assumption may not be realistic because on an *apriori* basis there is very little empirical evidence to choose one type of distribution over another.

Some researchers have proposed various fuzzy methods for dealing with the imprecision and ambiguity in DEA. Fuzzy set algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments. Sengupta (1992) proposed a fuzzy mathematical programming approach by incorporating fuzzy input and output data into a DEA model and defining tolerance levels for the objective function and constraint violations. Triantis and Giorod (1998) proposed a mathematical programming approach by transforming fuzziness into a DEA model using membership function values. Guo and Tanaka (2001), León et al. (2003) and Lertworasirikul et al. (2003a) proposed three similar fuzzy DEA...