Chapter 8
Sequential Test for Arbitrary Ratio of Mean Times between Failures

Yefim H. Michlin
Technion-Isreal Institute of Technology, Israel

Dov Ingman
Technion-Isreal Institute of Technology, Israel

Yoram Dayan
ELTA Systems Ltd., Israel

ABSTRACT

This paper describes a planning methodology and tools for a truncated SPRT (sequential probability ratio test) for checking the means ratio of the times between failures (assumed to be exponentially distributed) of two items. The problem is considered for the situation in which the ratio may differ from unity, whereby the results are applicable for any specified ratio, or wherever multiple copies of each item are tested simultaneously (group test). The authors present a methodology for optimal test choice and dependences for determining the acceptance/rejection boundaries of such a test with given characteristics. Planning and implementation of a group test are illustrated in an example, including substantiation of the choice of the number of new-item copies.

INTRODUCTION

The paper deals with development of a planning methodology and tools for truncated comparison SPRT testing of two items.

Such a comparison is effected, for example, in compliance (acceptance) testing, with a view to establishing that:

- A new item is superior/not inferior in reliability to a predecessor.
- A new production/repair technology for an item enhances/does not impair its reliability.

In simultaneous testing of the compared items the impact of changing ambient factors is elimi-
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nated or at least weakened, which is of special importance in accelerated tests.

Such comparison tests are typical not only in the field of reliability but also in many others, such as maintainability (Moustafa, 2008), efficacy assessment in decision making (Lagroue, 2008; Yeh & Yan, 2010), management (Xiao, Cai, & Jin, 2010).

The items in question are assumed to have exponentially distributed times to failure or times between failures (TBF), whose means are identically termed MTBF. One of these items is termed “new” (subscript $n$) and the other “basic” (subscript $b$), $\Phi$ being the ratio of their MTBF’s:

$$\Phi = \frac{MTBF_n}{MTBF_b} \quad (1)$$

The test consists in verifying that $\Phi$ is not less than a specified value $\Phi_0$:

$$\Phi \geq \Phi_0 \quad (2)$$

The exponential-TBF version is very common, as this distribution is the most characteristic in electronics and in complex devices (Kapur & Lamberson, 1977; Kececioglu, 1993; MIL-HDBK-781A, 1996; IEC-61124, 2006; HDBK-217-Plus, 2006). It is also practiced where groups of items are involved, thereby shortening the test duration (Epstein & Sobel, 1955).

Mace (1974) describes planning of such a test with fixed sample sizes (FSST). This version has the disadvantage of predetermined numbers of failures for both items – in which case if one item turns out greatly superior to the other, it will take very long for it to accumulate the necessary volume – whereas common sense would indicate an end to the test much sooner. Another disadvantage is the much larger sample sizes (or sample numbers – SN, see Wald (1947)) than the average requirement of the sequential probability ratio test (SPRT).

Girshick (1946) applies the SPRT theory in comparison tests of two items, the disadvantage here being the need for pairwise observation. A planning method for determining the scale-parameter ratio of two exponential distributions is proposed by Uno (2003). This problem differs from the one presented in this paper in that in our case it is not required to establish the actual value, but only that it is not less than a prescribed one. Uno’s solution calls for not less than two failures per item, which substantially impairs its efficacy when the scale parameters are widely different. The solution is approximative and has a significant bias. Zacks and Mukhopadhyay (2007) deal with an analogous problem of boundary construction and deciding on the MTBF ratio of two systems with exponential lifetimes. Their solution is exact, but their problem likewise differs from ours – namely, a ratio not less than a prescribed value.

In Michlin and Grabarnik (2007) the test under discussion is reduced to a binomial SPRT – one of the oldest described by Wald (1947). In Wald and Wolfowitz (1948) proof is provided of the optimality of the SPRT at two values of the estimated parameter – in this case $\Phi_0$ and $\Phi_1$, which in turn correspond to the null ($H_0$) and alternative ($H_1$) hypotheses respectively – and for which the operating characteristic (OC) has the values $(1-\alpha)$ and $\beta$. $\alpha$ and $\beta$ being the respective probabilities of type I and II errors. As for the optimality criterion, it is a minimal average SN (ASN) to stopping the test at given $\alpha$ and $\beta$.

The above two works list the following problems associated with SPRT planning:

a. Optimality at all values of the subject parameter.
b. Need for test truncation.
c. Determination of the OC and ASN from the given test boundaries.
d. Search for the boundaries for a given OC, acc. to the requirements for the statistical SN parameters.

As regards problem (a), Wald (1947, Section 2.3) states that the ASN function is the basis for