On Quasi Discrete Topological Spaces in Information Systems

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ABSTRACT

This paper presents an alternative way for constructing a topological space in an information system. Rough set theory for reasoning about data in information systems is used to construct the topology. Using the concept of an indiscernibility relation in rough set theory, it is shown that the topology constructed is a quasi-discrete topology. Furthermore, the dependency of attributes is applied for defining finer topology and further characterizing the roughness property of a set. Meanwhile, the notions of base and sub-base of the topology are applied to find attributes reduction and degree of rough membership, respectively.

Keywords: Indiscernibility Relation, Information System, Rough Set Theory, Topological Space, Topology

INTRODUCTION

In the 1980s, Z. Pawlak introduced rough set theory to deal the problem of imprecise knowledge (Pawlak, 1982). Similarly to fuzzy set theory (Zadeh, 1965), it is not an alternative to classical set theory but it is embedded in the set. Fuzzy and rough sets theories are not competitively, but complementary each other (Pawlak & Skowron, 2007). Rough set theory has attracted attention of many researchers and practitioners all over the world, who contributed essentially to its development and applications (Pawlak, 1983, 1997, 2002a, 2002b; Pawlak & Skowron, 2007; Herawan et al., 2009, 2010a, 2010b, 2010c, 2010d; Senan et al., 2011; Yanto et al., 2010a, 2010b, 2011, 2012). The original goal of the rough set theory is induction of approximations of concepts. The idea consists of approximation of a subset by a pair of two precise concepts called the lower approximation and upper approximation (Komorowski et al., 1999). Intuitively, the lower approximation of a set consists of all elements that surely belong to the set, whereas the upper approximation of the set constitutes of all elements that possibly belong to the set. The difference of the upper approximation and the lower approximation is a boundary region. It consists of all elements that cannot be classified uniquely to the set or its complement, by employing available knowledge. Thus any rough set, in contrast to a crisp set, has a non-empty boundary region. Motivation for rough set theory has come from the need to represent a subset of a universe in terms of equivalence classes of a partition of the universe.

The concept of topology shows up naturally in almost every branch of mathematics (Munk-
ers, 1975). This has made topology one of the great unifying ideas of mathematics. Rough set theory may therefore be considered as a method for constructing a topological space using indiscernibility relation on the universe. In a reverse process, we can generalize the notions of rough sets based on the topological space (Herawan & Deris, 2009). Definable sets, i.e. a union of one or more equivalence classes are substituted by open sets in defining the lower approximations, and by closed sets in defining upper approximations (Yao, 1998, 2001). Lashin et al. (2005) presented rough set theory in general topological space and described rough set theory in the topology of binary relation by defining (right) neighborhood and investigate the knowledge representations (granular structures) and processing of binary relations in the style of rough set theory. Zhu (2007) discussed covering-based rough sets from the topological view, defined a new type of covering-based rough sets from a topological concept called neighborhood and established axiomatic systems for the lower and the upper approximations operations. Allam et al. (2008) proposed and discussed some methods for generating topologies are using binary relations. They also investigated some properties of these topologies and obtained a quasi-discrete topology from a symmetric relation instead of an equivalence relation. Zhang et al. (2010) investigated the role of topological De Morgan algebra in the theory of rough sets.

However, in Lashin et al. (2005), Zhu et al. (2007), Allam et al. (2008), and Zhang et al. (2010), the concept of topological rough set in an information system is not discussed. Salama (2010) presented a new method of data decomposition to avoid the necessity of reasoning from data with missing attribute values. A general binary relation is defined on the original incomplete dataset. This binary relation generates data subsets without missing values. These data subsets are used to generate a topological base relation which decomposes datasets.

Instead of missing values in information system, in this paper, we apply “standard” rough set theory for topological space of reasoning from complete dataset (without missing attribute values). We begin with the using of rough set theory for constructing a topological space in information systems. The results show that: the family of all definable sets is a quasi-discrete topology on the universe, the partition induced by indiscernibility relation in information systems is the base for the related, and the union of partitions induced by singleton attribute is a sub-base for the related base. Further, we use the dependency of attributes in information systems for determining the finer topology and this can be used to characterize the roughness property of a set. Finally, we show that the attributes reduction and the degree of rough membership can be defined involving the sub-base and base for the related topology, respectively. Although some results are presented, a major part of this paper is devoted to reveal interconnections between rough set theory and topological spaces in information systems.

This paper is organized as follows. First, we describe the fundamental concepts of rough set theory for reasoning about data. Then a description of some notions in topological spaces. Afterwards we describe the main results, rough set theory for topological spaces in information systems. Finally, we conclude our work in the last section.

ROUGH SET THEORY

The observation that one cannot distinguish objects on the basis of given information about them is the starting point of rough set theory. In other words, imperfect information causes indiscernibility of objects. The indiscernibility relation induces an approximation space made of equivalence classes of indiscernible objects. A rough approximating a subset of the set of objects is a pair of dual approximation operators, called a lower approximation and an upper approximation in term of these equivalence classes (Yao, 1998). Rough sets are defined through their dual set approximations in Pawlak approximation space (Pawlak, 1982). Here, we use the concept of rough set theory in term of
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