Chapter 2
Propagation of Front Waves in Myelinated Nerve Fibres: New Electrical Transmission Lines Constituted of Linear and Nonlinear Portions

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ABSTRACT

In this paper, the authors examine the propagation of wave fronts in myelinated nerve fibres and applications as electrical transmission lines constituted of linear and nonlinear portions. Numerical simulations show that the front introduced in the nonlinear portion deforms itself in the linear portion, but recovers its initial profile and velocity in the next nonlinear portion. The phenomenon of deformation and recovery can be used for the development of new and low cost electrical transmission lines that can be used to transport localized excitations.

1. INTRODUCTION

An electrical line is an organ constituted of driver materials serving to the transport of energy or electricity. It can be linear or nonlinear. A nonlinear electrical line has particular importance in communication engineering. Indeed, some recent studies on electrical waves show that they can be interesting as optical wave in communication lines or even dominate these optical sibling since nonlinear electrical lines can be manufactured more easily (as compare to photonic devices) using standard integrated-circuit technology (Malomed, 1992; Hirota & Suzuki, 1970; Ricketts et al., 2006;
Lee, 2006; Nguimdo et al., 2008). A line constituted of alternated portions of linear and nonlinear portions could also serve for the propagation of nonlinear signals. Such an electrical line has as major advantage: the ease in the manufacturing of linear components and the reduction of the cost of the electrical line relatively to those constituted only of nonlinear components.

The aim of this study is first to understand the impulse propagation in myelinated nerve fibres considering the Ranvier nodes as nonlinear portions and the myelin sheath as linear portions. The second aim is to extend the results of the analysis to the propagation of front waves in electrical transmission lines with alternated linear and nonlinear portions. Particular attention is paid on the effects of the components of the linear portion on the profile and velocity of the wave.

Reutsky et al. (2003) presented a model of action potential propagation in myelinated nerve fibres. This model combines the single-cable formulation of Goldman and Albus (1968) with a basic representation of the ephaptic interaction among the fibres. The loss of the myelin sheath along tracts of central axons is observed in multi sclerosis, the most common demyelinating disease of central nervous system in humans (Smith et al., 1999). It has been experimentally demonstrated that demyelinated axons may become hyperexcitable and acquire the property of spontaneously generating trains of spurious impulses (Baker et al., 1992; Russell, 1982). The effects of demyelination and remyelination will be analyzed in this paper and its equivalent for electrical lines is the variation of electrical components of the linear portions.

The next section presents the propagation of impulses in myelinated nerve fibres. The propagation equations are derived and the numerical investigations are carried out in the case of constant electrical components and that of variable electric components. The effects of the disruption of the myelin sheath on the conduction velocity are analyzed. The suggestion of this model as electrical transmission lines with alternated linear and nonlinear portions is discussed. The conclusion appears in the last section.

2. EQUATION OF PROPAGATION

2.1 Description of the Model

Consider a myelinated nerve fibre as it appears in Figure 1a. In this figure, two consecutive Ranvier or active nodes are separated by a myelinated portion. The active node behaves as a nonlinear portion while the myelin sheath is electrically equivalent to a linear portion. The nonlinear and linear portions are represented by their electrical equivalents in Figure 1b for the Ranvier nodes and Figure 1c for the myelinated portion. \( R \) is the membrane resistance per unit length; \( C_1 \) and \( G_1 \) are the capacitance and conductance of the nonlinear portion while \( C_2 \) and \( G_2 \) represent the capacitance and conductance of the linear portion. The nonlinearity in a fibre resides in the conductance \( G_1 \) of the nonlinear portion as described below. To obtain equations (Equations 1-9) governing the propagation of the wave in any portion of the fibre, we apply Kirchhoff laws in Figure 1.

For the Ranvier nodes, the total transversal membrane current \( I \) is given by the following relation

\[
I = C_i \frac{dV}{dt} + I_i
\]

(1)

where \( V \) is the potential across the membrane and \( I_i \) is the ionic current through the ionic channels characterizing the movement of charged particles. Following Scott (1999), \( I_i \) is a nonlinear function of the potential \( V \) and is expressed as

\[
I = \frac{G_i}{V_i(V_b - V_d)} V(V - V_d)(V - V_b)
\]

(2)