Chapter 15
Superior Koch Curve

Sanjeev Kumar Prasad
Ajay Kumar Garg Engineering College, India

ABSTRACT

In this paper, the author presents the design of Superior Koch Curve with different scaling factor, which has wide applications in Fractals Graphics. The proposed curve has been designed using the technique of superior iteration. The Koch curve is the limiting curve obtained by applying the self similar divisions to infinite number of times but in Superior Koch Curve scaling factor is based on superior iteration.

1. INTRODUCTION

It is this similarity between the whole and its parts, even infinitesimal ones that make us consider this curve of von Koch as a line truly marvelous among all. If it were gifted with life, it would not be possible to destroy it without annihilating it whole, for it would be continually reborn from the depths of its triangles, just as life in the universe is. Koch Curve has wide application of Fractal Antenna (Fawwaz, 2008).

Recently, Rani and Kumar (2004) introduced superior iterations in the study of fractals and chaos and showed its power in a series of papers (Kumar et al., 2005, Negi et al., 2008, Rani et al., 2004, 2008). This paper was inspired by superior iterations (Rani et al., 2004) and created superior Koch Curves with different scaling factor.

2. PRELIMINARIES

Begin with a straight line (the blue segment in Figure 1). Divide it into three equal segments and replace the middle segment by the two sides of an equilateral triangle of the same length as the segment being removed (the two red segments in the figure). Now repeat, taking each of the four resulting segments, dividing them into three equal parts and replacing each of the middle segments
Superior Koch Curve

The Koch curve is the limiting curve obtained by applying this construction an infinite number of times. For a proof that this construction does produce a “limit” that is an actual curve, i.e., the continuous image of the unit interval, see the text by Edgar.

The first iteration for the Koch curve (Figure 2) consists of taking four copies of the original line segment, each scaled by $r = 1/3$. Two segments must be rotated by 60°, one counterclockwise and one clockwise. Along with the required translations, this yields the following Iterated Function System:

$$f_1(x) = \begin{bmatrix} .333 & 0 \\ 0 & .333 \end{bmatrix} x$$

Scale by $r$

$$f_2(x) = \begin{bmatrix} .167 & -.289 \\ .289 & 0.167 \end{bmatrix} x + \begin{bmatrix} .333 \\ 0 \end{bmatrix}$$

Scale by $r$, rotation by $-60°$

The fixed invariant set of this IFS (Iterated Function System) is same as the Koch curve.

2.1 Similarity Dimension

We have hyperbolic IFS (Iterated Function System) with each map being a similitude of ratio $r < 1$. Therefore the similarity dimension, $d$, of the unique invariant set of the IFS is the solution to

$$\sum_{i=1}^{4} r^d = 1 \implies d = \frac{\log(1/4)}{\log r} = \frac{\log 4}{\log 3} = 1.2619$$

Figure 1. Koch curve

Figure 2. First iteration of Koch curve
Related Content

Content, Context & Connectivity Persuasive Interplay
[www.igi-global.com/article/content-context--connectivity-persuasive-interplay/100455?camid=4v1a](www.igi-global.com/article/content-context--connectivity-persuasive-interplay/100455?camid=4v1a)

Noise Power Spectrum for Firecrackers
[www.igi-global.com/chapter/noise-power-spectrum-firecrackers/75918?camid=4v1a](www.igi-global.com/chapter/noise-power-spectrum-firecrackers/75918?camid=4v1a)

Signs Conveying Information: On the Range of Peirce’s Notion of Propositions: Dicisigns
[www.igi-global.com/article/signs-conveying-information/56446?camid=4v1a](www.igi-global.com/article/signs-conveying-information/56446?camid=4v1a)

A Hybrid System for Automatic Infant Cry Recognition II
[www.igi-global.com/chapter/hybrid-system-automatic-infant-cry/10345?camid=4v1a](www.igi-global.com/chapter/hybrid-system-automatic-infant-cry/10345?camid=4v1a)