Computing the Risk Indicators in Fuzzy Systems

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ABSTRACT

The modeling of complex risk situations imposes the existence of multiple ways to represent the risk and compare the risk situations between them. In probabilistic models, risk is described by random variables and risk situations are compared by stochastic dominance. In possibilistic or credibilistic models, risk is represented by fuzzy variables. This paper concerns three indicators of dominance associated with fuzzy variables. This allows the definition of three notions of fuzzy dominance: dominance in possibility, dominance in necessity and dominance in credibility. These three types of dominance are possibilistic and credibilistic versions of stochastic dominance. Each type offers a modality of ranking risk situations modeled by fuzzy variables. In the paper some properties of the three indicators of dominance are proved and relations between the three types of fuzzy dominance are established. For triangular fuzzy numbers formulas for the computation of these indicators are obtained. The paper also contains a contribution on a theory of risk aversion in the context of credibility theory. Using the credibilistic expected utility a notion of risk premium is defined as a measure of risk aversion of an agent in front of a risk situation described by a fuzzy variable and an approximate calculation formula of this indicator is proved.

Keywords: Credibilistic Computation of Risk Aversion, Credibilistic Risk Premium, Fuzzy Dominance, Fuzzy Variable, Risk in Fuzzy Systems, Stochastic Dominance

1. INTRODUCTION

Risk is a phenomenon which accompanies many economic and social activities. By Diamond and Stiglitz (1974, p. 337), there are two fundamental topics in the analysis of risk situations:

1. What “a situation is riskier than another” means;
2. What “one individual is more risk averse than another” means.

Usually the study of risk situations is done by probabilistic methods (Arrow, 1970; Mas-Colell, Whinston & Green, 1995; Rothschild & Stiglitz, 1970; Wolfstetter, 2002). On the other hand, the complexity of economic and social phenomena leads also to risk situations for which probability theory does not offer a faithful description any more (e.g. when we do not have huge amounts of data). Another modality to study risk situations is Zadeh’s (1978) possibility theory. This falls into the mainstream known as “fuzzy sets and systems” initiated by

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In possibility theory risk situations are modeled by possibility distributions (among which the fuzzy numbers) and the analysis of risk is done by possibilistic indicators (possibilistic expected value, possibilistic variance and covariance, etc).

Georgescu (2011) is an attempt of possibilistic approach to topic (b). In Georgescu (2011) a possibilistic model of risk aversion of an agent confronted by a situation of uncertainty described by a fuzzy number is proposed. This model is based on a theory of possibilistic indicators elaborated by Carlsson, Fullér & Majlender (2001, 2003, 2005).

Several authors started to treat issue (a) from the perspective of possibility theory (e.g. Aiche & Dubois, 2010). Credibility theory initiated by Liu & Liu (2002) and then developed axiomatically in monograph (Liu, 2007) is an instrument stronger than possibility theory in the study of situations of uncertainty. Research (Peng, Monk, & Tse, 2005; Peng, Liu, & Shang, 2006; Peng & Jiang, 2007; Zuo & Ji, 2009) deal with issue (a) using credibility theory. The results of these papers can be regarded as credibilistic versions of the results of Diamond and Stieglitz (1974), Fishburn (1980), and Rothschild and Stieglitz (1970).


The second part of the paper studies the issue (b) in the context of credibility theory.

The paper is organized as follows.

In Section 2 fuzzy numbers and some of their properties are recalled. Section 3 presents the notions of possibility measure, necessity measure and credibility measure and the bijective correspondences which can be established between these entities. Section 4 is dedicated to the three indexes of dominance and some properties of possibilistic dominance, dominance in necessity and dominance in credibility are studied. In Section 5 the indexes of dominance of triangular fuzzy numbers are computed and some properties of the three types of dominance in this particular case are proved.

In Liu and Liu (2002) with each discrete fuzzy variable a discrete random variable is canonically associated. Section 6 studies the relation between the credibilistic dominance of discrete fuzzy variables and stochastic dominance of associated discrete random variables. First one proves that the indexes of credibilistic dominance of a discrete fuzzy variable equal the indexes of stochastic dominance of the associated discrete fuzzy variable. This result is later used to establish that in this case the credibilistic dominance turns into stochastic dominance.

Section 7 contains exemplifications of the results of Section 6. The paper concludes with an application which recalls the definitions of the indexes of credibilistic dominance and stochastic dominance of order n.

In Section 8 two notions of credibilistic risk premium are defined as indicators of the risk aversion of an agent in front of risk represented by a fuzzy variable. The credibilistic element which appears in their definition is the use of credibilistic expected utility from Liu (2007). Relations between the two indicators are established and approximate calculation formulas are proved.

2. FUZZY NUMBERS

In this section we recall from Dubois and Prade (1980), Zadeh (1965), Zimmerman (1984) some definitions and elementary properties of fuzzy numbers.
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